TRIANGULAR FUZZY GRACEFUL LABELING IN SOME PATH RELATED GRAPHS

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Abstract. In this paper, we introduce a new labeling technique called Triangular Fuzzy Graceful Labeling. We prove the fuzzy union and fuzzy intersection of two paths $p_m, p_n, (m, n \leq 30)$ admits Triangular Fuzzy Labeling.

1. Introduction

In 20th century, remarkable development had happened in mathematical modeling for uncertainty which was introduced by Lofti. A. Zadeh in 1965[10] called fuzzy sets. The applications of fuzzy sets in the field of cluster analysis, neural networks etc were discussed by Zimmermann [11]. Zadeh [10] had developed the fuzzy relations on fuzzy sets which had better feature in making fuzzy graph model.

In 1973 Kauffman[8] first introduced the concept of fuzzy graphs and then it had been developed by Azirel Rosedfield [9]. Many results on fuzzy graphs were proved by J.N. Moderson, K.R. Bhutani, A. Rosenfield, S. Mathew and M. Sunita[1, 2, 3, 9]. A. Nagoorani, D. Rajalakshmi(a) Subhasini, introduced fuzzy graph labeling and studied such as graceful labeling, vertex graceful labeling in some special graphs, etc. S. Vimala and R. Jebesty Shajila has studied the edge vertex graceful labeling [7].

In this paper, we introduce a new fuzzy labeling technique called Triangular Fuzzy Graceful Labeling and prove some results.

2. Preliminaries

Definition 1. A fuzzy set is defined mathematically by assigning to each possible individual in the universe of discourse, a value representing its grade of membership in the fuzzy sets.

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Definition 2. A fuzzy number is a fuzzy set on the real line that satisfies normality and convexity. One of the standard types of fuzzy number called Triangular Fuzzy number (TFN) which has the membership function

\[ \mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{x-a_3}{a_2-a_3}, & a_2 \leq x \leq a_3 \\ 0, & \text{elsewhere.} \end{cases} \]

Example 3. \( \tilde{A} = (2, 3, 4) \) is a triangular fuzzy number.

Definition 4. A graph \( G = (V, E) \) consists of a nonempty set \( V \) called set of vertices and a set \( E \) of ordered or unordered pairs of elements of \( V \) called the set of edges.

Definition 5. A fuzzy graph \( G = (\sigma, \mu) \) is a pair function \( \sigma : V \rightarrow [0, 1] \) and \( \mu : V \times V \rightarrow [0, 1] \) where for all \( u, v \in V \), we have

\[ \mu(u, v) \leq \sigma(u) \wedge \sigma(v). \]

Definition 6. A path \( P \) in a fuzzy graph is a sequence of distinct nodes \( v_1, v_2, ..., v_n \) such that

\[ \mu(v_i, v_{i+1}) > 0, \ 1 \leq i \leq n. \]

Definition 7. A labeling of a graph is on assignment of values to the vertices and edges subject to certain conditions.

Definition 8. A graceful labeling of a graph \( G \) with \( q \) edges is an injection \( f : V(G) \rightarrow \{0, 1, 2, ..., q\} \) such that when each edge \( xy \in E(G) \) is assigned the label \( |f(x) - f(y)| \), all the edge labels are distinct.

Definition 9. A fuzzy graph \( G = (\sigma, \mu) \) is said to be fuzzy labeling graph if \( \sigma : V \rightarrow [0, 1] \) and \( \mu : V \times V \rightarrow [0, 1] \) is bijective and

\[ \mu(u, v) < \sigma(u) \wedge \sigma(v), \ for \ all \ u, v \in V. \]

Definition 10. A fuzzy labeling in which all the edge(vertex) values are distinct is called graceful fuzzy labeling. A graph which admits a fuzzy graceful labelling is called fuzzy graceful graph.
3. TRIANGULAR FUZZY LABELING GRAPH

Definition 11. A fuzzy labeling on $G$ is said to be a triangular fuzzy labeling if the labels are assigned to the vertices are triangular fuzzy number.

Here triangular fuzzy number is a fuzzy number which is a subset of $[0, 1]$ denoted by $TFN$ that is $TFN \subseteq [0, 1]$.

Example 12.

$$
\begin{align*}
\sigma : V \rightarrow TFN, \quad \sigma(v_i) &= \frac{i^2}{100}, \forall v_i \in V, 1 \leq i \leq n \\
\mu : V \times V \rightarrow TFN, \quad \mu(v_{i+1}, v_i) &= |\sigma(v_{i+1}) - \sigma(v_i)|, \forall v_i \in V, 1 \leq i \leq n.
\end{align*}
$$

From the above assumption, we have

$$
\sigma(v_i) > \sigma(v_{i+1}), \quad \text{and} \quad \mu(v_{i+1}, v_i) > \mu(v_{i+2}, v_{i+1})
$$

clearly $G_1$ is a Triangular Fuzzy Labeling Graph.

Definition 13. Union of two fuzzy labeled graph is a fuzzy labeled graph with

$$
G_1 \cup G_2 = \{\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2\}
$$

where $G_1 : (\sigma_1, \mu_1)$ with $\sigma_1 : V_1 \rightarrow [0, 1], \mu_1 : V_1 \times V_1 \rightarrow [0, 1], \forall v_i \in V_1, 1 \leq i \leq m,$ and $G_2 : (\sigma_2, \mu_2)$ with $\sigma_2 : V_2 \rightarrow [0, 1], \mu_2 : V_2 \times V_2 \rightarrow [0, 1], \forall v_j \in V_2, 1 \leq j \leq n.$

Example 14.
4. UNION OF TRIANGULAR FUZZY GRACEFUL LABELING IN SOME PATH RELATED GRAPHS

**Theorem 15.** The path graph $p_m, (m \leq 30)$ admits fuzzy triangular graceful labeling.

**Proof.** Let $p_m = (\sigma_1, \mu_1)$ be a path graph. Define an injective map $\sigma : V \rightarrow TFN$ such that

$$\sigma(v_i) = \frac{m^2 + (i^2 - \frac{1}{100})}{1000}, \quad 1 \leq i \leq m.$$ 

The edge values of $p_m$ is calculated by defining a function $\mu : \times V \rightarrow [0, 1]$ such that

$$\mu_1(v_i, v_{i+1}) = |\sigma(v_{i+1}) - \sigma(v_i)|, \quad \forall v_i \in V.$$ 

Clearly $\mu_1(v_i, v_{i+1})$ is distinct for all the edges of $p_m$. Hence $p_m$ is a triangular fuzzy graceful graph. $\square$

**Theorem 16.** The fuzzy union of fuzzy triangular labeled path graph is also a fuzzy triangular labeled path graph.

**Proof.** Let path graphs $p_m, p_n, (m, n \leq 30)$ be two fuzzy triangular labeled path graph.

Consider the path $p_m : (\sigma_1, \mu_1)$ with

1. $\sigma_1 : V_1 \rightarrow TFN \ni \sigma_1(v_i) = \frac{(m)^2 + (i^2 - \frac{1}{100})}{1000}, \forall v_i \in V_1, 1 \leq i \leq m$

2. and $\mu_1 : V_1 \times V_1 \rightarrow TFN \ni \mu_1(v_{i+1}, v_i) = |\sigma_1(v_{i+1}) - \sigma(v_i)|$.

Another path $p_n : (\sigma_2, \mu_2)$ with

3. $\sigma_2 : V_2 \rightarrow TFN \ni \sigma_2(v_j) = \frac{j^2}{10000}, \forall v_j \in V_2, 1 \leq j \leq n$
and

(4) \( \mu_2 : V_2 \times V_2 \rightarrow TFN \ni \mu_2(v_{j+1}, v_j) = |\sigma_2(v_{j+1}) - \sigma(v_j)|. \)

By definition \( p_m \cup p_n = \{\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2\} \). Now

(5) \( (\sigma_1 \cup \sigma_2)(V_i V_j) = \max\{\sigma_1(v_i), \sigma_2(v_j)\} = \sigma_1(v_i) \)

since by (1), (3) \( \sigma_1(v_i) > \sigma_2(v_j) \)

(6) \( (\mu_1 \cup \mu_2)((v_{i+1}, v_i)(v_{j+1}, v_j)) = \max\{\mu_1(v_{i+1}, v_i), \mu_2(v_{j+1}, v_j)\} \)

since by (2), (4) \( \mu_1(v_{i+1}, v_i) > \mu_2(v_{j+1}, v_j) \). Hence by (5) and (6), \( p_m \cup p_n \) is clearly fuzzy triangular labeled path graphs. \( \square \)

**Example 17.**

Clearly \( p_m \cup p_n \) is a fuzzy triangular labeled path graphs.

**Example 18.**

Clearly \( p_m \cup p_n \) is a fuzzy triangular labeled path graphs.
Theorem 19. The fuzzy union of fuzzy triangular graceful path graphs need not be a fuzzy triangular graceful path graph.

Proof. Let $p_m, p_n, (m, n \leq 30)$ be two fuzzy triangular graceful path graphs. From Theorem 16

$$p_m \cup p_n = \{\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2\}$$

$$= \{\sigma_1(v_i), \mu_1(v_{i+1}, v_i)\} \ \forall v_i \in V_1, \ 1 \leq i \leq m.$$

Clearly $p_m \cup p_n$ be the triangular fuzzy labeled path graphs and it does not admit distinct edge labeling and distinct vertex labeling. This shows that $p_m \cup p_n$ need not be a fuzzy triangular graceful labeled path graphs. □

Example 20.

$p_m \cup p_n$ does not admit distinct edge and distinct vertex labeling. Hence $p_m \cup p_n$ need not be a fuzzy triangular graceful labeled path graphs.
5. Intersection of Triangular Fuzzy Graceful Labeling in graph related graphs

Definition 21. Intersection of two fuzzy labeled graphs is a fuzzy labeled graph with

\[ G_1 \cap G_2 = \{ \sigma_1 \cap \sigma_2, \mu_1 \cap \mu_2 \} \]
\[ = \{ (\sigma_1 \cap \sigma_2)(vi,vj), (\mu_1 \cap \mu_2)(v_{i+1},v_i)(v_{j+1},v_j) \} \]
\[ = \{ \text{min}\{\sigma_1(v_i), \sigma_2(v_j)\}, \text{min}\{\mu_1(v_{i+1},v_i), \mu_2(v_{j+1},v_j)\} \} \]

where \( G_1 : (\sigma_1, \mu_1) \) with \( \sigma_1 : V_1 \rightarrow [0, 1], \mu_1 : V_1 \times V_1 \rightarrow [0, 1], \forall v_i \in V_1, 1 \leq i \leq m \) and \( G_2 : (\sigma_2, \mu_2) \) with \( \sigma_2 : V_2 \rightarrow [0, 1], \mu_2 : V_2 \times V_2 \rightarrow [0, 1], \forall v_j \in V_2, 1 \leq j \leq n \).

Example 22.
Theorem 23. The fuzzy intersection of fuzzy triangular labeled path graph is also a fuzzy triangular labeled path graph.

Proof. Let $p_m, p_n, (m, n \leq 30)$ be two fuzzy triangular labeled path graph.
Consider the path $p_m : (\sigma_1, \mu_1)$ with
\begin{align*}
\sigma_1 : V_1 \to TFN & \ni \sigma_1(v_i) = \frac{(m)^2 + (i^2 - 1/100)}{1000}, \forall v_i \in V_1, 1 \leq i \leq m \\
\mu_1 : V_1 \times V_1 & \to TFN \ni \mu_1(v_{i+1}, v_i) = |\sigma_1(v_{i+1}) - \sigma_1(v_i)|.
\end{align*}
and
\begin{align*}
\sigma_2 : V_2 \to TFN & \ni \sigma_2(v_j) = \frac{j^2}{10000}, \forall v_j \in V_2, 1 \leq j \leq n \\
\mu_2 : V_2 \times V_2 & \to TFN \ni \mu_2(v_{j+1}, v_j) = |\sigma_2(v_{j+1}) - \sigma(v_j)|.
\end{align*}
By definition
\[ p_m \cap p_n = \{\sigma_1 \cap \sigma_2, \mu_1 \cap \mu_2\}. \]
Now
\[ (\sigma_1 \cap \sigma_2)(v_iv_j) = \min\{\sigma_1(v_i), \sigma_2(v_j)\} = \sigma_2(v_j) \]
(11) since by (7),(9) $\sigma_2(v_j) < \sigma_1(v_i)$, and
\[ (\mu_1 \cap \mu_2)(v_{i+1}, v_i)(v_{j+1}, v_j) = \min\{\mu_1(v_{i+1}, v_i), \mu_2(v_{j+1}, v_j)\} = \mu_2(v_{j+1}, v_j) \]
(12) since by (8),(10) $\mu_2(v_{j+1}, v_j) < \mu_1(v_{i+1}, v_i)$. Hence by (11) and (12), $p_m \cap p_n$ is clearly fuzzy triangular labeled path graphs. \qed

Example 24.

Clearly $p_m \cap p_n$ is a fuzzy triangular labeled path graph.
Theorem 25. The fuzzy intersection of triangular graceful path graphs need not be a fuzzy triangular graceful path graph.

Proof. Let $p_m, p_n, (m, n \leq 30)$ be two fuzzy triangular graceful path graphs. From Theorem 23,

$$p_m \cap p_n = \{ \sigma_1 \cap \sigma_2, \mu_1 \cap \mu_2 \}$$

$$= \{ \sigma_2(v_j), \mu_2(v_{j+1}, v_j) \} \ \forall v_j \in V_2, 1 \leq j \leq n.$$ 

Clearly $p_m \cap p_n$ be the triangular fuzzy labeled path graphs and it does not admit distinct edge labeling and distinct vertex labeling. Which leads that $p_m \cap p_n$ need not be a fuzzy triangular graceful labeled path graphs.

Remark 1. Example 24 shows that $p_m \cap p_n$ does not admit distinct labeling in vertices and edges and clearly $p_m \cap p_n$ need not be a fuzzy triangular graceful path graph.

6. Conclusion

A new labeling technique called triangular fuzzy graceful labeling has been introduced. The operations like fuzzy union and fuzzy intersection of triangular fuzzy labeled path graphs have been discussed. Further research work to be extended for some special graphs.

References


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