On magnetohydrodynamic peristaltic pumping of an incompressible viscous fluid with chemical reactions and wall properties

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Abstract

This paper addresses the influence of wall properties, simultaneous homogeneous-heterogeneous chemical reactions and applied magnetic field on an incompressible viscous fluid with peristalsis. Fluid is considered in a channel which is inclined and symmetric. Involved problem is analyzed through long wavelength hypothesis and conditions of Taylor's limit. The expression for the mean effective coefficient of dispersion is developed and physically interpreted through graphical illustrations.

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1 Introduction

Peristalsis is the most important method of transporting numerous physiological liquids. It is utilized by many systems in the living body to push or to mix up the substances of a hose. This mechanism is used in some biomedical devices: hose pumps, finger and roller pumps that is used it to force blood, slurries, and other fluids. In the view of its importance, several experts ([1],[2]) have examined the peristaltic transport of different liquids under various circumstances.

The dispersion of a solute in a solvent flowing in a channel has applications in physiological fluid dynamics, biomedical and chemical engineering. The basic theory on dispersion was first proposed by Taylor [3], who investigated the viscous incompressible laminar flow of a fluid in a circular pipe with dispersion of solute matter. Author believes that, the solute disperses with an equivalent average effective dispersion coefficient, and the dispersion depended on the radius of the tube, coefficient of molecular diffusion and average speed of the flow. Gupta and Gupta [4], Sobh [5], Chandra and Philip [6] investigated the dispersion of a solute in viscous fluid under different limitations. In addition, these analyses have been extended to non Newtonian fluids by [7].

The main results are presented in concluding section. We hope that the present results have relevance in process like cancer therapy, chemical industry, transport of targeted drug using magnetic field, and the transport of water, nutrients to various branches of tree and moment of nutrients in blood veins.

2 Two-dimensional fluid flow model

Consider the magneto-hydrodynamic peristaltic pumping of an incompressible viscous fluid in the 2- dimensional inclined channel. The peristaltic wave produces the flow travelling along the channel walls. Figure 1 describes the geometry of the problem.

\[ y = \pm h = \pm \left[ d + a \sin \frac{2\pi}{\lambda} (x - ct) \right] \]  
where \( a \) - the amplitude, \( c \) - the speed and \( \lambda \) - the wavelength of the peristaltic wave, and \( d \) - the half width of the channel.
The corresponding flow equations of the present issue are as follows:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \] (2)

\[ \rho \left[ \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right] u = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) u - \sigma B_0^2 u + \rho g \sin \theta, \] (3)

\[ \rho \left[ \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right] v = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) v - \rho g \cos \theta, \] (4)

where \( \rho \)-density, \( p \)-pressure, \( \mu \)-viscosity coefficient, \( u, v \)-velocity component in the \( x, y \) direction, \( g \)-gravity due to acceleration, \( \theta \)-angle of inclination, \( B_0 \)-magnetic field.

The equation of the bendable wall movement ([1]) is given as:

\[ L(h) = p - p_0 \] (5)

where \( L \)- the operator which is used to represent movement of an expanded membrane by the damping forces:

\[ L = -T \frac{\partial^2}{\partial x^2} + m \frac{\partial^2}{\partial t^2} + C \frac{\partial}{\partial t} \] (6)

Here \( m \)- the mass per/area \( T \)- the tension in the membrane, and \( C \)- the viscous damping force coefficient.

After solving the equations (2) to (4) under long-wavelength hypothesis, we get

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \] (7)

\[ -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u + \rho g \sin \theta = 0, \] (8)

\[ \frac{\partial p}{\partial y} = 0 \] (9)

The related periphery conditions are

\[ u = 0 \text{ at } y = \pm h \] (10)

It is presumed that \( p_0 = 0 \) and the channel walls are inextensible; therefore, the horizontal displacement of the wall is zero and only
lateral movement takes place.

\[
\frac{\partial}{\partial x} L(h) = \mu \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u + \rho g \sin \theta \quad \text{at} \quad y = \pm h \quad (11)
\]

where

\[
\frac{\partial}{\partial x} L(h) = \frac{\partial p}{\partial x} = -T \frac{\partial^3 h}{\partial x^3} + m \frac{\partial^3 h}{\partial x \partial t^2} + c \frac{\partial^2 h}{\partial x \partial t} = P' \text{say} \quad (12)
\]

After solving the equations (8) to (9) with conditions (10) and (11), we get

\[
u(y) = \frac{A'}{\mu m_1^2} \left[ \cosh(m_1 y) \cosh(m_1 h) - 1 \right] \quad (13)
\]

The mean velocity is given as

\[
\bar{u} = \frac{1}{2h} \int_{-h}^{h} u(y) dy = \frac{A'}{\mu m_1^2} \left[ \frac{\sinh(m_1 h)}{m_1 h \sinh(m_1 h)} - 1 \right] \quad (14)
\]

Utilizing [18], the fluid velocity is given by the equation:

\[
u_x = u - \bar{u} = \frac{A'}{\mu m_1^2} [L'_1 \cosh(m_1 y) - L'_2] \quad (15)
\]

where,

\[
L'_1 = \frac{1}{\cosh(m_1 h)}, \quad L'_2 = \frac{\sinh(m_1 h)}{m_1 h \cosh(m_1 h)}, \quad A' = \frac{\partial p}{\partial x} - \frac{\rho g}{\mu} \sin \theta, \quad m_1 = \sqrt{\frac{\sigma B_0^2}{\mu}}
\]

**Diffusion of simultaneous homogeneous and heterogeneous chemical Reactions**

Following [3] and [4], the dispersion equation for the concentration \(C\) of the substance for the present issue under isothermal conditions:

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial y^2} - k_1 C \quad (16)
\]

In the above equation, \(C\) - concentration of the fluid, \(D\) - the molecular diffusion coefficient and \(k_1\) - the rate constant of first order
chemical reaction. Utilizing $\tilde{u} = C$ of [7] and consequent non-dimensional quantities,

$$\theta = \frac{t}{\tilde{t}}, \bar{t} = \lambda \tilde{u}, \eta = \frac{y}{d}, \xi = \frac{(x - \bar{u}t)}{\lambda}, H = \frac{h}{\lambda}, P = \frac{d^2}{\mu c} P', \gamma = \frac{\rho g}{\mu}. \quad (17)$$

Equations (12), (15) and (16) reduce to

$$P = -\epsilon \left[ (E_1 + E_2)(2\pi)^3 \cos(2\pi \xi) - E_3(2\pi)^2 \sin(2\pi \xi) \right] \quad (18)$$

$$u_x = \frac{d^2}{\mu m^2} A \left[ L_1 \cosh(m\eta) - L_2 \right] \quad (19)$$

$$\frac{\partial^2 C}{\partial \eta^2} - \frac{k_1 d^2}{D} C = \frac{d^2}{\lambda D} u_x \frac{\partial C}{\partial \xi} \quad (20)$$

where $A = P - \gamma \sin \theta$, $L_1 = \frac{1}{\cosh mH}$, $L_2 = \frac{\sinh mH}{mH \cosh mH}$, $\epsilon = \frac{a}{d}$ is the amplitude ratio, $E_1 = -\frac{d^3}{m^3 \mu}$ is the rigidity, $E_2 = \left( \frac{m c d}{\lambda \mu} \right)$ is the stiffness, $E_3 = \left( \frac{c d^3}{\mu \lambda^2} \right)$ is the viscous damping force in the wall.

The diffusion with first-order irreversible chemical reaction taking place in the mass of the fluid medium and at the walls of the channel is discussed and treated that the walls are catalytic to chemical reaction referring [6] for the periphery conditions at the walls and is given as:

$$\frac{\partial C}{\partial y} + f C = 0 \quad \text{at} \quad y = h = [d + a \sin \frac{2\pi}{\lambda} (x - \bar{u}t)] \quad (21)$$

$$\frac{\partial C}{\partial y} - f C = 0 \quad \text{at} \quad y = -h = -[d + a \sin \frac{2\pi}{\lambda} (x - \bar{u}t)] \quad (22)$$

From equations (17), (21) and (22), we get

$$\frac{\partial C}{\partial \eta} + \beta C = 0 \quad \text{at} \quad \eta = H = [1 + \epsilon \sin(2\pi \xi)] \quad (23)$$

$$\frac{\partial C}{\partial \eta} - \beta C = 0 \quad \text{at} \quad \eta = -H = [-1 + \epsilon \sin(2\pi \xi)] \quad (24)$$

where $\beta = f d$ is the heterogeneous reaction rate parameter corresponding to catalytic reaction at the walls.
From equations (23), (24) the primitive of equation (20) as follows:

\[ C(\eta) = \left[ d^4 \lambda \mu D m^2 \frac{\partial C}{\partial \xi} \right] A \left[ L_1 \cosh (m \eta) - L_5 \cosh (\alpha \eta) + L_6 - L_7 \cosh (\alpha \eta) \right] \]  

(25)

and \( \alpha = \sqrt{\frac{k_1}{D}} \), \( m = m_1 d = \sqrt{\frac{\sigma}{\mu}} B_0 d = M \) say

The volumetric rate \( Q \) is defined as the rate in which the solute is pumping across a section of channel per unit breadth.

\[ Q = \int_{-H}^{H} C u_x d\eta \]  

(26)

Using equations (19) and (25) in equation (26), we obtain

\[ Q = -2 \frac{d^6}{\lambda \mu^2 D} \frac{\partial C}{\partial \xi} G(\xi, \alpha, \beta, \epsilon, E_1, E_2, E_3, M, \gamma, \theta) \]  

(27)

where

\[ G(\xi, \alpha, \beta, \epsilon, E_1, E_2, E_3, M, \gamma, \theta) = \]

\[ - \frac{A^2}{m^4} \left[ \frac{L_1 L_4}{2} L_8 - (L_1 L_5 + L_4 L_7)L_9 + (L_1 L_6 - L_2 L_4)L_{10} + (L_2 L_5 + L_7 L_6)L_{11} - L_2 L_6 H \right] \]  

(28)

Looking at equation (28) with Ficks law of diffusion, the scattering coefficient \( D^* \) was calculated such that the solute diffuses comparative to the plane moving with the average speed of the flow and is given as:

\[ D^* = 2 \frac{d^6}{\mu^2 D} G(\xi, \alpha, \beta, \epsilon, E_1, E_2, E_3, M, \gamma, \theta) \]  

(29)

Let \( \bar{G} \) be the average of \( G \) and is obtained by the following equation:

\[ \bar{G} = \int_{0}^{1} G(\xi, \alpha, \beta, \epsilon, E_1, E_2, E_3, M, \gamma, \theta) d\xi \]  

(30)

3 Illustrations and Discussion

The expression for \( \bar{G}(\xi, \alpha, \beta, \epsilon, E_1, E_2, E_3, M, \gamma, \theta) \) has been obtained by numerical integration using the software MATHEMATICA and
end results are presented through graphs. We may ensure that $E_1, E_2$ and $E_3$ cannot be zero all together. We have considered the figures 2-4 for the effect of the rigidity parameter ($E_1$), stiffness ($E_2$) and viscous damping force ($E_3$) on the dispersion coefficient $G$. It is observed that $G$ ascends monotonically with an increase in $E_1$, $E_2$ and $E_3$. These results are in agreement with the results of Ravikiran and Radhakrishnamacharya [7]. Figures 5-6 indicates that $G$ descences with an increase in the magnetic parameter ($M$). This finding agrees with the conclusion of Sobh [5]. From figures 7-8, it is noticed that, $G$ amplify as angle of inclination ($\theta$) increases. These end results agrees with the outcomes of Sankad and Radhakrishnamacharya [2]. Dispersion reduces with homogeneous compound response rate
parameter ($\alpha$) (Figures 3, 4, 6, 8) and heterogeneous substance response rate ($\beta$) (Figures 2, 5, 7), where as scattering diminishing with $\beta$ is less significant. This outcome is normal since expansion in prompts an expansion in number of moles of solute experiences chemical response. This result consistent with the arguments of Gupta and Gupta [4].

4 Main findings

It is observed from the previous section that, concentration profile $\bar{G}$ ascends with an increase in wall parameters ($E_1, E_2, E_3$), inclination parameter($\theta$) and amplitude ratio ($\epsilon$). Furthermore, opposite behaviors of homogeneous response rate parameter ($\alpha$) and hetero-
geneous response rate parameter $\beta$ are observed on $\bar{G}$. Finally, it concludes that inclination parameter, wall parameters and amplitude ratio favor the dispersion, while magnetic parameters resist the dispersion in the digestive framework.

References


