PERISTALTIC TRANSPORT OF A PSEUDOPLASTIC FLUID BOUNDED BY PERMEABLE WALLS WITH SUCTION AND INJECTION

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February 9, 2017

Abstract

This paper deals with the peristaltic transport of a Pseudoplastic fluid bounded by the permeable walls with suction and injection. The expressions for velocity, pressure rise and the frictional force has been evaluated by the perturbation analysis in the wave frame of reference under the assumptions of long wavelength and low Reynolds number. The variation of the pressure rise and frictional force with the flux over one wavelength are shown graphically. It is observed that the higher the suction/injection parameter, the smaller the pressure rise against the pump works.

AMS Subject Classification: 76D05, 76A05, 11Y35

Keywords: pseudoplastic fluid; suction and injection; permeable wall, perturbation.

1 Introduction

The investigation on the peristaltic pumping has been attracting attention of biomechanical engineers because of its importance in both physiological and mechanical situations. In recent years, Vajravelu et al. [1], Reddy et al. [2] and Kavitha [3] made detailed investigations on the peristaltic pumping of Newtonian (or) non-Newtonian fluid in ducts of different cross sections and peristaltic flows with suction and injection has been studied in [4-6]. Noreen et al. [7, 8] made detailed analysis on the peristaltic flow of Pseudoplastic fluid in a symmetric and an asymmetric channel.

In view of these, we study the peristaltic transport of a Pseudoplastic fluid bounded by permeable walls with suction and injection in a symmetric channel under the assumptions of low Reynolds number and long wavelength.


2 Mathematical Formulation

Consider the peristaltic transport of a Pseudoplastic fluid bounded by permeable walls of the channel of width $2a$. The fluid is injected into the channel perpendicular to the lower permeable wall with a constant velocity $V_0$ and is sucked out of the upper permeable wall with the same velocity $V_0$ as shown in the Fig.1. For simplicity, we restrict our discussion to the half width of the channel. The wall deformation is given by

$$ Y = H(X, t) = a + b \cos \frac{2\pi}{\lambda} (X - Ct) $$

(1)

where $b$ is the amplitude, $\lambda$ is the wave length and $c$ is the wave speed.

\[\text{Figure 1: Physical Model}\]

The transformations from fixed frame to wave frame are introduced as follows:

$$ x = X - ct, y = Y, u = U - c, v_0 = V_0, p(x, y) = P(X, Y, t) $$

(2)

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced

$$ \bar{x} = \frac{x}{\lambda}, \bar{y} = \frac{y}{a}, \bar{u} = \frac{u}{c}, \bar{v}_0 = \frac{v_0}{c}, \delta = \frac{a}{\lambda}, \bar{t} = \frac{ct}{\lambda}, \bar{p} = \frac{\rho u c \lambda p}{a c}, \phi = \frac{b}{a}, \text{Re} = \frac{ca\rho}{\mu}, \mu_1 = \frac{\mu c}{a}, $$

$$ \bar{S}_{ij} = \frac{\alpha S_{ij}}{\mu c} (\text{for } i, j = 1, 2, 3..., \lambda_1 = \frac{\lambda c}{a}, \beta = \frac{\beta}{a}, u = \frac{\partial \psi}{\partial y}, v = -\delta \frac{\partial \psi}{\partial x} ) $$

(3)

Here $\delta$, Re are the wave and Reynolds numbers respectively; $\beta$ is the slip parameter, $\psi$ is the stream function, $\lambda_1$ and $\mu_1$ are the non-dimensional relaxation times.

Under the assumptions of long wavelength and low Reynolds number (after dropping bars),
we get

\[
\frac{\partial p}{\partial x} = \frac{\partial S_{xy}}{\partial y} - k \frac{\partial u}{\partial y} \quad (4)
\]

\[
\frac{\partial p}{\partial y} = 0 \quad (5)
\]

where \( k = Re.v_0; \)

\[
S_{xx} = (\lambda_1 + \mu_1) S_{xy} \left( \frac{\partial u}{\partial y} \right); \quad S_{yy} = (-\lambda_1 + \mu_1) S_{xy} \left( \frac{\partial u}{\partial y} \right); \quad S_{xy} = \frac{\partial u/\partial y}{1 + \xi (\partial u/\partial y)^2}.
\]

The corresponding dimensionless boundary conditions in wave frame of reference are given by

\[
\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (6)
\]

\[
u = -1 - \beta \frac{\partial u}{\partial y} \quad \text{at} \quad y = h = 1 + \phi \cos 2\pi x \quad (7)
\]

The volume flow rate \( q \) in a wave frame of reference is given by

\[
q = \int_0^{h(x)} u \, dy \quad (8)
\]

The instantaneous flux \( Q(X, t) \) in a fixed frame is

\[
Q(X, t) = \int_0^h U \, dY = \int_0^h (u + 1) \, dy = q + h \quad (9)
\]

The time average flux \( \bar{Q} \) over one period \( T (= \lambda/c) \) of the peristaltic wave is

\[
\bar{Q} = \frac{1}{T} \int_0^T Q \, dt = \int_0^1 (q + h) \, dx = q + 1. \quad (10)
\]

3 \textbf{Perturbation Solution}

The equation (4) is non-linear. we linearize this equation in terms of \( \xi \), the small relaxation parameter for the flow. So we expand \( \mu, p \) and \( q \) as

\[
\begin{align*}
\{ u & = u_0 + \xi u_1 + O(\xi^2) \\
p & = p_0 + \xi p_1 + O(\xi^2) \\
q & = q_0 + \xi q_1 + O(\xi^2) \}
\end{align*}
\]
Substituting (11) in the equation (4) and in the boundary conditions (6) and (7) and equating the coefficients of like powers of $\xi$ to zero and neglecting the terms of $\xi^2$ and higher order, we get the following equations:

3.1 Equation of order $\xi^0$

\[
\frac{dp_0}{dx} = \frac{\partial^2 u_0}{\partial y^2} - k \frac{\partial u_0}{\partial y}
\]

(12)

and the respective boundary conditions are

\[
\frac{\partial u_0}{\partial y} = 0 \quad \text{at} \quad y = 0
\]

(13)

\[
u_0 = -1 - \beta \frac{\partial u_0}{\partial y} \quad \text{at} \quad y = h
\]

(14)

3.2 Equation of order $\xi^1$

\[
\frac{dp_1}{dx} = \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial}{\partial y} \left[ \left( \frac{\partial u_0}{\partial y} \right)^3 \right] - k \frac{\partial u_1}{\partial y}
\]

(15)

and the respective boundary conditions are

\[
\frac{\partial u_1}{\partial y} = 0 \quad \text{at} \quad y = 0
\]

(16)

\[
u_1 = -\beta \frac{\partial u_1}{\partial y} \quad \text{at} \quad y = h
\]

(17)

Solving the equation (12) by using the boundary conditions (13) and (14), we get

\[
u_0 = -1 + p_0 \left( k_1 + \frac{e_k y}{k^2} - \frac{y}{k} \right)
\]

(18)

where $k_1 = \frac{h}{k} - \frac{e_k h}{k^2} + \frac{\beta}{k} - \frac{e_k h}{k}$

and the volume flow rate $q_0$ is given by

\[
q_0 = \int_0^h u_0 \, dy = P_0 k_3 - h
\]

(19)

where $k_3 = k_1 h + \frac{e_k h}{k^2} - \frac{h^2}{2k} - \frac{1}{k^2}$

From equation (20), we get

\[
\frac{dp_0}{dx} = p_0 = \frac{(q_0 + h)}{k_3}
\]

(20)
Solving the equation (15) by using the equation (18) and the boundary conditions (16) and (17), we get
\[ u_1 = P_1 \left[ k_1 + \frac{e^{ky}}{k^2} - \frac{y}{k} \right] + P_0^3 \left[ k_2 + \frac{3 e^{ky}}{2 k^4} + \frac{3}{k^2} \left[ \frac{e^{3ky}}{6k^2} - \frac{e^{2ky}}{k^2} + \frac{y}{k} e^{ky} \right] \right] \]  \hspace{1cm} (21)

where
\[ k_2 = \left[ -\frac{3}{2} e^{kh} - \frac{3}{2} e^{kh} + \frac{3}{k^2} \left[ \frac{e^{3kh}}{6k^2} - \frac{e^{2kh}}{k^2} + \frac{h e^{kh}}{k} \right] - \frac{3\beta}{k^2} \left[ \frac{e^{3kh}}{2k^2} - 2 e^{2kh} + \frac{17}{6} \right] \right] \]

and the volume flow rate \( q_1 \) is given by
\[ q_1 = \int_0^h u_1 \, dy = P_1 k_3 + P_0^3 k_4 \]  \hspace{1cm} (22)

where \( k_4 = k_2 h + \frac{3h e^{kh}}{k^4} + \frac{1}{k^2} \left[ \frac{3e^{kh}}{2} + \frac{e^{3kh}}{6} - \frac{3e^{2kh}}{2} - 3eh^{kh} + \frac{17}{6} \right] \)

From equations (22), we have
\[ \frac{dp_1}{dx} = q_1 k_3 - P_0^3 k_4 \]  \hspace{1cm} (23)

Substituting equations (20) and (23) into the equation (11) and using the relation
\[ \frac{dp}{dx} = \frac{dp_1}{dx} - \xi \frac{dp_1}{dx} \]  \hspace{1cm} and neglecting terms greater than \( O(\xi^2) \) we get
\[ u = -1 + P \left[ k_1 + \frac{e^{ky}}{k^2} - \frac{y}{k} \right] + \xi P^3 \left[ k_2 + \frac{3 e^{ky}}{2 k^4} + \frac{3}{k^2} \left[ \frac{e^{3ky}}{6k^2} - \frac{e^{2ky}}{k^2} + \frac{y}{k} e^{ky} \right] \right] \]  \hspace{1cm} (24)

Similarly,
\[ \frac{dp}{dx} = \frac{(q + h)}{k_3} - \xi (q + h)^3 \frac{k_4}{(k_3)^4} \]  \hspace{1cm} (25)

The dimensionless pressure rise and frictional force per one wavelength in the wave frame are defined, respectively as
\[ \Delta p = \int_0^1 \frac{dp}{dx} \, dx \]  \hspace{1cm} (26)

and
\[ F = \int_0^1 h \left( -\frac{dp}{dx} \right) \, dx. \]  \hspace{1cm} (27)
4 Results and Discussion

From Fig. 2, we notice that in the pumping region ($\Delta P > 0$), the larger the suction parameter ($k$) the smaller the pressure rise against which the pump works. In Fig. 3, we observe that the larger the permeability parameter $\beta$ the smaller the pressure rise against which the pump works. From Fig. 4, we observe that the larger the $\xi$, the smaller the pressure rise against which the pump works. Finally from equation (28), we have calculated frictional force $F$ as a function of $\bar{Q}$ and the graph of variation are depicted in Fig. 5. It is observed that the frictional force $F$ has opposite behaviour compared to pressure rise $\Delta P$.

![Figure 2: The variation of $\Delta P$ with $\bar{Q}$ for $k$ with $\phi=0.5$, $\beta=0.4$, $\xi=0.01$](image)

5 Conclusion

In this paper, the peristaltic transport of a Pseudoplastic fluid bounded by permeable walls with suction and injection under the assumptions of long wavelength and low Reynolds number. The pressure rise and frictional force per wavelength are computed by numerical integration. The main observations are as follows:

1. Increase in the suction/injection parameter $k$ and the permeability parameter $\beta$ gives rise to decrease in the pressure rise.

2. Increase in the small perturbation parameter $\xi$ leads to decrease the pressure rise
Figure 3: The variation of $\Delta P$ with $\bar{Q}$ for $\beta$ with $\phi=0.5$, $\xi=0.01$, $k=0.9$

Figure 4: The variation of $\Delta P$ with $\bar{Q}$ for $\xi$ with $\phi=0.5$, $k=0.9$, $\beta=0.4$

Figure 5: The variation of $F$ with $\bar{Q}$ for $k$ with $\phi=0.5$, $\beta=0.4$, $\xi=0.01$
References


