Abstract

In this paper, we assign membership function for edges of a graph so that the graph becomes a fuzzy regularizable graph. Then we develop graph structures for these fuzzy regularizable graphs and study its properties.

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Key Words and Phrases: Graph structures, Wheel graph, Double Wheel graph, Triple Wheel graph, Vertex and Edge Cohesive number.

1 Introduction

The concept of Fuzzy sets was introduced by L.A.Zadeh in 1965. Graph structure concept was introduced by E.Sampath kumar (2006). Fuzzy graph structures were introduced by Ramakrishnan and T.Dinesh (2011). Fuzzy regularizable graph was introduced by B.Poornima and V K Ramaswamy[4]. These concepts have applications in communication networks.

Preliminaries

Let, \( V \) be a non-empty set. A fuzzy graph [3] is a function \( G(\sigma, \mu) \) where \( \sigma : V \rightarrow [0, 1] \) and \( \mu : V \times V \rightarrow [0, 1] \) such that \( \mu(u, v) \leq \)
$\sigma(u) \land \sigma(v) \forall u, v \text{ in } V$.

A graph structure [11] $G = (V, R_1, R_2 \ldots R_k)$ consists of a non-empty set $V$ together with relations $R_1, R_2 \ldots R_k$ on $V$ which are mutually disjoint such that each $R_i$, $1 \leq i \leq k$, is symmetric and irreflexive. A set $S$ of vertices in $R_1, R_2 \ldots R_k$ structure $G$ is $R_i$-cohesive for some $i$, $1 \leq i \leq k$, if $S$ is $R_i$ - connected. The vertex cohesive number $C_v(G)$ of a graph structure $G = (V, R_1, R_2 \ldots R_k)$ is the minimum order of a partition of $V$ into cohesive sets. The edge cohesive number $C_e(G)$ of $G$ is the minimum order of a partition of the edge set $E$ of $G$ into cohesive sets.

**Examples**

**Example 1.** [11] Let the graph $G(V, E)$ be

$V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$

The graph structure $G(V, R_1, R_2)$ is

$V = \{v_1, v_2, v_3, v_4\}$, $R_1 = \{(v_2, v_3), (v_3, v_4), (v_4, v_1)\}$

$R_2 = \{(v_1, v_2)\}$

Now, Consider the graph structure, $G(V, R_1, R_2)$ (Fig. 1)

![Figure 1](image1.jpg)

Figure 1: $C_v(G) = 1$

Here $c(v_1) = 2$, $c(v_2) = 2$, $c(v_3) = 1$, $c(v_4) = 1$.

![Figure 2](image2.jpg)

Figure 2: $C_v(G) = 1$

![Figure 3](image3.jpg)

Figure 3: $C_e(G) = 2$

**Methodology:** A fuzzy graph from a crisp graph is developed. Fuzzy graph structure, concept is developed by grouping the edges
with same membership function. We observe that it satisfies the property of fuzzy regularizable graph [4]. Hence we get a fuzzy regularizable graph structure and study its properties.

2 Fuzzy Regularizable Graph Structures

A graph is fuzzy regularizable if the sum of membership of edges incident on every vertex is same.

**Theorem 2.** For a fuzzy wheel regularizable graph structure the vertex cohesive number and edge cohesive number is 1 and 2.

**Proof.** Consider $W_n(C_n + K_1$ is the wheel with $n + 1$ vertices and $2n$ edges) (Fig. 4)

![Figure 4:](image)

Here, degree of $v = n$. Degree of $v_i = 3, i = 1$ ton.

Let $\mu(v, v_i) = \frac{r}{n}, i = 1$ ton ($r \leq n$)

Sum of membership of edges incident on

$$v = \frac{r}{n} + \frac{r}{n} + \cdots + \frac{r}{n}, \quad n \text{ times} = n \cdot \frac{r}{n} = r.$$  

$$\mu(v_i, v_{i+1}) = \frac{nr - r}{2n}, \quad i = 1 \text{ to } n - 1.$$  

$$\mu(v_n, v_i) = \frac{nr - r}{2n}$$

Sum of membership of edges incident on

$$v_i = \frac{r}{n} + \frac{nr - r}{2n} + \frac{nr - r}{2n} = r.$$
$W_n$ is a fuzzy regularizable graph. Now, group the edges with same edge membership values.

Say, $R_1 = \{(v, v_1), (v, v_2), \ldots, (v, v_n)\}$

$R_2 = \{(v_1, v_2), (v_2, v_3), \ldots, (v_{n-1}, v_n), (v_n, v_1)\}$

$(W_n, R_1, R_2)$ is a fuzzy regularizable graph structure (Fig 5).

The vertex cohesive number of $(W_n, R_1, R_2)$ is 1, Since (Fig. 6)

The edge cohesive number of $(W_n, R_1, R_2)$ is 2, Since (Fig. 6 & Fig. 7) For a fuzzy wheel regularizable graph structure, the vertex cohesive number and edge cohesive number is 1 and 2.

**Theorem 3.** For a fuzzy double wheel regularizable graph structure, the vertex cohesive number and edge cohesive number is 1 and 3.

**Proof.** Consider $DW_n(2C_n + K_1$ is called double wheel with $2n + 1$ vertices and $4n$ edges) (Fig. 8)

Here, degree of $v = 2n$. Degree of $v_i = 3$, $i = 1$ to $n$.

Degree of $w_i = 3$, $i = 1$ to $n$. Let $\mu(v, v_i) = \frac{r}{2n}$, $i = 1$ to $n$ ($r \leq 2n$)
\[ \mu(v, w_i) = \frac{r}{2n}, \quad i = 1 \text{ to } n \quad (r \leq 2n) \]

Sum of membership of edges incident on

\[ v = \frac{r}{2n} + \frac{r}{2n} + \cdots + \frac{r}{2n}, \quad 2n \text{ times} = 2n \cdot \frac{r}{2n} = r. \]

\[ \mu(v, v_{i+1}) = \frac{2nr - r}{4n}, \quad i = 1 \text{ to } n - 1. \]

\[ \mu(v, v_1) = \frac{2nr - r}{4n} \]

Sum of membership of edges incident on

\[ v_i = \frac{r}{2n} + \frac{2nr - r}{4n} + \frac{2nr - r}{4n} = r. \]

\[ \mu(w_i, w_{i+1}) = \frac{2nr - r}{4n}, \quad i = 1 \text{ to } n - 1. \]

\[ \mu(w_n, w_1) = \frac{2nr - r}{4n} \]

Sum of membership of edges incident on

\[ w_i = \frac{r}{2n} + \frac{2nr - r}{4n} + \frac{2nr - r}{4n} = r. \]

\(DW_n\) is a fuzzy regularizable graph. Now, group the edges with same edge membership values.

Let, \( R_1 = \{ (v, v_1), (v, v_2), \ldots, (v, v_n), (v, w_1), (v, w_2), \ldots, (v, w_n) \} \)

\( R_2 = \{ (v_1, v_2), (v_2, v_3), \ldots, (v_{n-1}, v_n), (v_n, v_1), (w_1, w_2), (w_2, w_3), \ldots, (w_{n-1}, w_n), (w_n, w_1) \} \)

\((DW_n, R_1, R_2)\) is a fuzzy regularizable graph structure (Fig 9). The vertex cohesive number of \((DW_n, R_1, R_2)\) is 1, Since (Fig. 10).

![Figure 10:](image1)

![Figure 11:](image2)

The edge cohesive number of \((DW_n, R_1, R_2)\) is 3, Since (Fig. 10 & Fig. 11). For a fuzzy double wheel regularizable graph structure the vertex cohesive number and edge cohesive number is 1 and 3.
**Theorem 4.** For a fuzzy triple wheel regularizable graph structure, the vertex cohesive number and edge cohesive number is 1 and 4.

**Proof.** Consider $TW_n(3C_n+K_1)$ is called double wheel with $3n+1$ vertices and $6n$ edges (Fig. 12).

Here, degree of $v = 3n$. Degree of $v_i = 3$, $i = 1$ ton. Degree of $u_i = 3$, $i = 1$ ton.

Let $\mu(v, v_i) = \frac{r}{3n}$, $i = 1$ ton ($r \leq 3n$)

$\mu(v, w_i) = \frac{r}{3n}$, $i = 1$ ton ($r \leq 3n$), $\mu(v, u_i) = \frac{r}{3n}$, $i = 1$ ton ($r \leq 3n$)

Sum of membership of edges incident on

$$v = \frac{r}{3n} + \frac{r}{3n} + \cdots + \frac{r}{3n}, \quad 3n \text{ times} = 3n, \frac{r}{3n} = r.$$

$$\mu(v, v_{i+1}) = \frac{3nr - r}{6n}, \quad i = 1 \text{ to } n - 1.$$

$$\mu(v_n, v_1) = \frac{3nr - r}{6n}$$

Sum of membership of edges incident on

$$v_i = \frac{r}{3n} + \frac{3nr - r}{6n} + \frac{3nr - r}{6n} = r.$$

$$\mu(w_i, w_{i+1}) = \frac{3nr - r}{6n}, \quad i = 1 \text{ to } n - 1.$$

$$\mu(w_n, w_1) = \frac{3nr - r}{6n}$$
Sum of membership of edges incident on $w_i = r 3n + 3nr - r 6n + 3nr - r 6n = r$. 

$\mu(u_i, u_{i+1}) = \frac{3nr - r}{6n}, \quad i = 1 \text{ to } n - 1.$

$\mu(u_n, u_1) = \frac{3nr - r}{6n}$

Sum of membership of edges incident on $u_i = r 3n + 3nr - r 6n + 3nr - r 6n = r$.

$TW_n$ is a fuzzy regularizable graph. Now, group the edges with same edge membership values.

Let, $R_1 = \{(v, v_1), (v, v_2), \ldots, (v, v_n), (v, w_1), (v, w_2), \ldots, (v, w_n), (v, u_1), (v, u_2), \ldots, (v, u_n)\}$

$R_2 = \{(v_1, v_2), (v_2, v_3), \ldots, (v_{n-1}, v_n), (v_n, v_1), (w_1, w_2), (w_2, w_3), \ldots, (w_{n-1}, w_n), (w_n, w_1), (u_1, u_2), (u_2, u_3), \ldots, (u_{n-1}, u_n), (u_n, u_1)\}$

$(TW_n, R_1, R_2)$ is a fuzzy regularizable graph structure (Fig 13). The vertex cohesive number of $(TW_n, R_1, R_2)$ is 1, Since (Fig. 14).

![Figure 14](image1.png) ![Figure 15](image2.png)

The edge cohesive number of $(TW_n, R_1, R_2)$ is 4, Since (Fig. 14 & Fig. 15). For a fuzzy triple wheel regularizable graph structure the vertex cohesive number and edge cohesive number is 1 and 4. □

In general, following theorem can be proved.

**Theorem 5.** For a fuzzy $KW_n$ regularizable graph structure, the vertex cohesive number and edge cohesive number is 1 and $K + 1$. 
Example for a fuzzy graph structure which is not a fuzzy regularizable graph structure

Any fuzzy graph structure which has pendant edges is not a fuzzy regularizable graph structure. (Sum of the membership function incident on a pendant vertex is not equal to the sum of the membership function of a non-pendant vertex).

3 Conclusion

The membership functions for edges are assigned for wheel graphs which becomes a fuzzy regularizable graph. Then we develop graph structures for these fuzzy regularizable graphs and study its properties.

References


