STRONGLY IRREGULAR INTERVAL VALUED FUZZY GRAPHS

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Abstract: Theoretical concepts of a graphs are highly utilized by computer science applications. Especially in research areas of computer science such as data mining, image segmentation, clustering, image capturing and networking. The interval-valued fuzzy graphs are more flexible and compatible than fuzzy graphs due to the fact that they allowed the degree of membership of a vertex to an edge to be represented by the interval values in [0, 1] rather than the crisp values between 0 and 1. In this paper, we introduced strongly irregular interval valued fuzzy graph and strongly total irregular interval valued fuzzy graph. The relation between strongly, highly and neighbourly irregular interval valued fuzzy graphs are established. Also defined domination in irregular fuzzy graph.

Key Words: degree of fuzzy graph, regular fuzzy graph, irregular fuzzy graph, highly, neighbourly, strongly, strongly total irregular fuzzy graph, highly irregular interval valued fuzzy graph, strongly, strongly total irregular interval valued fuzzy graph and ∆-domination

1. Introduction

and Ayyaswamy [4] suggested a method to construct a neighbourly irregular graph of order n and also discussed some properties on neighbourly irregular graph. Yousef Alavi, et al., [12] introduced k-path irregular graph and studied some properties on k-path irregular graphs. Nagoor Gani and Latha [8] introduced neighbourly irregular fuzzy graphs, neighbourly total irregular fuzzy graphs, highly irregular fuzzy graphs and highly total irregular fuzzy graphs. SP. Nandhini and E. Nandhini [9] introduced strongly irregular fuzzy graphs, strongly total irregular fuzzy graphs. Madhumangal Pal and Hossein Rashmanlou [5] introduced Irregular interval-valued fuzzy graphs. In this paper, we define a strongly irregular interval valued and strongly total irregular interval valued fuzzy graphs comparative study between strongly irregular interval valued and strongly total irregular interval valued fuzzy graphs are made. Some results on strongly irregular interval valued fuzzy graphs and strongly total irregular fuzzy graphs interval valued are studied. Also defined delta domination in irregular fuzzy graph. Throughout this paper only undirected fuzzy graphs are considered.

2. Preliminaries

A fuzzy set $A$ on a set $X$ is characterized by a mapping $m : X \rightarrow [0, 1]$ called the membership function.

A fuzzy set is denoted as $A = (X, m)$. A fuzzy graph [10] $\xi = (V, \sigma, \mu)$ is a non-empty set $V$ together with a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$ such that for all $u, v \in V$, $\mu(u, v) \leq \sigma(u) \land \sigma(v)$

{here $x \land y$ denotes the minimum of $x$ and $y$}. Partial fuzzy graph $\xi' = (V, \tau, \mu')$ of $\xi$ is such that $\tau(v) \leq \sigma(v)$ for all $v \in V$, and $\mu(u, v) \leq \mu'(u, v)$ for all $u, v \in V$. Fuzzy graph $\xi' = (P, \sigma', \mu')$ of $\xi$ is such that $P \subseteq V$ $\sigma'(v) = \sigma(v)$ for all $u \in P$ and $\mu'(u, v) = \mu(u, v)$ for all $u, v \in P$.

Let $X$ be a nonempty set. An interval-valued fuzzy sets $A$ in $V$ is defined by $A = \{x, [\mu_A^-, \mu_A^+] : x \in V\}$ where $\mu_A^-(x)$ and $\mu_A^+(x)$ are fuzzy subsets of $V$ such that $\mu_A^-(x) \leq \mu_A^+(x)$ for all $x \in V$.

If $G^* = (V, E)$ is a graph, then by an interval valued fuzzy relation $B$ on a set $E$, i.e. an interval valued fuzzy set such that $\mu_B^-(xy) \leq \min(\mu_A^-(x), \mu_A^-(y))$, $\mu_B^+(xy) \leq \min(\mu_A^+(x), \mu_A^+(y))$ for all $xy \in E$.

An interval-valued fuzzy graph [5] of a graph $G^* = (V, E)$, a pair $G = (A, B)$, where $A = [\mu_A^-, \mu_A^+]$ is an interval valued fuzzy set on $V$ and $B = [\mu_B^-(x), \mu_B^+(x)]$ is an interval valued fuzzy relation on $E$.

The graph $G$ is called Complete interval-valued fuzzy graph if
\( \mu_{B^{-}}(xy) = \min(\mu_{A^{-}}(x), \mu_{A^{-}}(y)) \), \( \mu_{B^{+}}(xy) = \min(\mu_{A^{+}}(x), \mu_{A^{+}}(y)) \) for all \( xy \in V \).

The Complement of an interval -valued fuzzy graph \[ G = (A, B) \] of a graph \( G^* = (V, E) \) is called an interval-valued fuzzy graph \( G^* = G = (\bar{A}, \bar{B}) \) of \( \bar{G}^* = (\bar{V}, \bar{V} \times \bar{V}) \) where \( \bar{A} = A = [\mu_{A^{-}}, \mu_{A^{+}}] \) and \( \bar{B} = [\bar{\mu}_{B^{-}}, \bar{\mu}_{B^{+}}] \) is defined by \( \bar{\mu}_{B^{-}}(xy) = \mu_{A^{-}}(x) \wedge \mu_{A^{-}}(y) - \mu_{B^{-}}(xy) \) for all \( x, y \in V \) and \( \bar{\mu}_{B^{+}}(xy) = \mu_{A^{+}}(x) \wedge \mu_{A^{+}}(y) - \mu_{B^{+}}(xy) \) for all \( x, y \in V \).

Let \( G = (A, B) \) be an interval-valued fuzzy graph where \( A = [\mu_{A^{-}}, \mu_{A^{+}}] \) and \( B = [\mu_{B^{-}}, \mu_{B^{+}}] \) be two interval-valued fuzzy sets on a non-empty finite set \( V \) and \( E \subseteq V \times V \) respectively. The positive degree of a vertex \( u \in G \) is \( d^{+}(u) = \sum_{uv \in E} \mu_{B^{+}}(uv) \). Similarly, the negative degree of a vertex \( u \in G \) is \( d^{-}(u) = \sum_{uv \in E} \mu_{B^{-}}(uv) \). The degree of a vertex \( u \) is \( d(u) = [d^{-}(u), d^{+}(u)] \). If \( d^{+}(u) = k_1, d^{+}(u) = k_2 \) for all \( u \in V \), \( k_1, k_2 \) are two real numbers, then the graph is called \([k_1, k_2]\)-regular interval valued fuzzy graph.

Let \( G = (A, B) \) be an interval-valued fuzzy graph where \( A = [\mu_{A^{-}}, \mu_{A^{+}}] \) and \( B = [\mu_{B^{-}}, \mu_{B^{+}}] \) be two interval-valued fuzzy sets on a non-empty finite set \( V \) and \( E \subseteq V \times V \) respectively. \( G \) is said to be irregular interval-valued fuzzy graph if there exists a vertex which is adjacent to a vertex with distinct degrees.

Let \( G = (A, B) \) be an interval-valued fuzzy graph where \( A = [\mu_{A^{-}}, \mu_{A^{+}}] \) and \( B = [\mu_{B^{-}}, \mu_{B^{+}}] \) be two interval-valued fuzzy sets on a non-empty finite set \( V \) and \( E \subseteq V \times V \) respectively. The total degree of the vertex \( u \in V \) is denoted by \( td(u) \) and is defined as \( td(u) = [td^{-}(u), td^{+}(u)] \) where \( td^{+}(u) = \sum_{uv \in E} \mu_{B^{+}}(u, v) + \mu_{A^{+}}(u), td^{-}(u) = \sum_{uv \in E} \mu_{B^{-}}(u, v) + \mu_{A^{-}}(u) \). If the total degrees of all vertices of an interval valued fuzzy graph are equal then the graph is said to be totally regular interval valued fuzzy graph. \( G \) is said to be totally irregular interval-valued fuzzy graph if there exists a vertex which is adjacent to a vertex with distinct total degrees.

### 3. Properties of Strongly irregular interval valued fuzzy graphs

**Definition 3.1.** [3] Let \( G = (A, B) \) be a connected interval valued fuzzy graph. \( G \) is said to be a highly irregular interval valued fuzzy graph if every vertex of \( G \) is adjacent to vertices with distinct neighborhood degrees.

**Definition 3.2.** [5] Let \( G = (A, B) \) be a connected fuzzy graph. \( G \) is said to be a neighbourly irregular interval valued fuzzy graph if every two adjacent vertices of \( G \) have distinct degree.

**Definition 3.3.** Let \( G = (A, B) \) be a connected interval valued fuzzy graph. \( G \) is said to be a strongly irregular interval valued fuzzy graph if every pair of vertices in \( G \) have distinct degrees.
Example 3.4.

\[ d(u) = (0.2, 0.4), \quad d(v) = (0.3, 0.5), \quad d(w) = (0.5, 0.7), \quad d(x) = (0.4, 0.6) \]

**Theorem 3.5.** If \( G = (A, B) \) is a strongly irregular interval valued fuzzy graph then it is both highly irregular interval valued fuzzy and neighbourly interval valued irregular fuzzy graph.

**Proof:** Obvious

**Theorem 3.6.** A highly irregular interval valued fuzzy graph and neighbourly irregular interval valued fuzzy graph \( G = (A, B) \) need not be a strongly irregular interval valued fuzzy graph.

**Proof:** Suppose \( u \) and \( v \) be any two vertices of \( G \), which are not adjacent and not incident on same vertex may happen to have same degrees. This contradicts the definition of Strongly irregular interval valued fuzzy graph.

\[ d(u) = (0.2, 0.7), \quad d(w) = (0.1, 0.4), \quad d(x) = (0.1, 0.4), \quad d(v) = (0.6, 1.1) \]

Here \( d(x) = d(w) \)

**Theorem 3.7.** Let \( G = (A, B) \) highly irregular interval valued fuzzy and neighbourly irregular interval valued fuzzy graph. If every pair of vertices in \( G \) is either adjacent or incident on the same vertex then \( G \) is strongly irregular interval valued.

**Proof:** Obvious

**Theorem 3.8.** A fuzzy graph \( G = (A, B) \) where \( G^* \) is a cycle with
vertices 3 is strongly irregular interval valued if and only if the weights of the
dges between every pair of vertices are all distinct.

Proof: For, if the weights of any edges are the same , it violates the
definition of strongly irregular interval valued fuzzy graphs.
Conversely, the weights of edges between every pair of vertices are all distinct.
Let u, v and w are the vertices of G
Suppose $d^+(u) = d^+(v)$

$\implies \mu_{B^+}(u,v) + \mu_{B^+}(u,w) = \mu_{B^+}(u,v) + \mu_{B^+}(v,w)$

$\implies \mu_{B^+}(u,w) = \mu_{B^+}(v,w)$, a contradiction.
Similarly we can prove for negative degree.
Therefore G is Strongly irregular interval valued fuzzy graph.

Theorem 3.9. The complement of a strongly irregular interval valued
fuzzy graph need not be strongly irregular interval valued.

Proof:Obvious

Theorem 3.10. Let $G = (A, B)$ be a interval valued fuzzy graph , where
$G^*$ is regular, $\mu_{A^+}$ and $\mu_{A^-}$ is a constant function and $\mu_{B^+(-)}(u,v) <$
$\mu_{A^+(-)}(u) \land \mu_{A^+(-)}(v)$ for all $u, v \in V(G)$.Then G is a strongly irregular interval valued fuzzy graph iff $G^c$ is a strongly irregular interval valued fuzzy graph

Proof: Let $G = (\sigma, \mu)$ be a strongly irregular interval valued fuzzy graph
and $\sigma(u) = c$ for all $u \in G$.
$d^+(u) \neq d^+(v)$ for all $u, v \in V(G)$.

$\Leftrightarrow \sum \mu_{B^+}(u, x_i) \neq \sum \mu_{B^+}(v, y_j) \forall x_i$ incident on $u \forall y_j$ incident on $v$

$\Leftrightarrow \sum [c - \mu_{B^+}(u, x_i)] \neq \sum [c - \mu_{B^+}(v, y_j)] \forall x_i$ incident on $u \forall y_j$ incident on $v$, since $G^*$ is regular.

$\Leftrightarrow \sum [\mu_{A^+}(u) \land \mu_{A^+}(x_i) - \mu_{B^+}(u, x_i)] \neq \sum [\mu_{A^+}(v) \land \mu_{A^+}(y_i) - \mu_{B^+}(v, y_i)] \forall x_i$

$\Leftrightarrow \sum \mu_{B^+}(u, x_i) \neq \sum \mu_{B^+}(v, y_j) \forall u, v \in G^c$.
Similarly we can prove for the negative degree.
Hence G is strongly irregular interval valued fuzzy graph.

Theorem 3.11. Let $G = (A, B)$ be a strongly irregular interval valued
fuzzy graph then the partial fuzzy subgraph $H = (A', B')$ need not be strongly
irregular interval valued fuzzy graph

Proof:Obvious

Theorem 3.12. The fuzzy subgraph $H = (A', B')$ of a strongly irregular
interval valued fuzzy graph $G = (A, B)$ is strongly irregular interval valued
fuzzy graph.
Proof: Let $G = (A, B)$ be a strongly irregular interval valued fuzzy graph.

$\Rightarrow d^+(u) \neq d^+(v)$ for all $u, v \in V(G)$.

$\Rightarrow \sum \mu_B^+(u, x_i) \neq \sum \mu_B^+(v, y_j) \ \forall x_i$ incident on $u$ & $\forall y_j$ incident on $v$

$\Rightarrow d^+(u) \neq d^+(v)$ for all $u, v \in V(H)$

**Theorem 3.13.** The underlying crisp graph of a fuzzy graph $G=(A, B)$ is complete then $G$ is a neighbourly irregular interval valued fuzzy graph if and only if $G$ is a strongly irregular interval valued fuzzy graph.

**Proof:** Let the underlying crisp graph of a fuzzy graph $G=(A, B)$ is complete. Suppose $G$ is neighbourly irregular interval valued fuzzy graph $\iff$ Every two vertices are adjacent and have distinct degrees $\iff G$ is a strongly irregular interval valued fuzzy graph.

**Theorem 3.14.** The underlying crisp graph of a fuzzy graph $G=(A, B)$ is complete then $G$ is a highly irregular interval valued fuzzy graph if and only if $G$ is a strongly irregular interval valued fuzzy graph.

**Proof:** Let the underlying crisp graph of a fuzzy graph $G=(A, B)$ is complete with $n$ vertices. Every vertex of $G$ is adjacent to remaining $(n-1)$ vertices. Suppose $G$ is highly irregular interval valued fuzzy graph $\iff$ every vertex of $G$ is adjacent to vertices with distinct degrees $\iff$ every vertices of $G$ have distinct degrees $\iff G$ is a strongly irregular interval valued fuzzy graph.

4. Properties Of Strongly Total Irregular interval valued Fuzzy Graph:

**Definition 4.1.** Let $G = (A, B)$ be a connected interval valued fuzzy graph. $G$ is said to be a highly total irregular interval valued fuzzy graph if every vertex of $G$ is adjacent to vertices with distinct neighborhood total degrees.

**Definition 4.2.** Let $G = (A, B)$ be a connected fuzzy graph. $G$ is said to be a neighbourly total irregular interval valued fuzzy graph if every two adjacent vertices of $G$ have distinct total degrees.

**Definition 4.3.** Let $G = (A, B)$ be a connected fuzzy graph. $G$ is said to be a strongly total irregular interval valued fuzzy graph if every pair of vertex in $G$ have distinct total degrees.

**Example 4.4.** In example 2.4 $td(u)=(0.7, 1)$, $td(v)=(0.6, 0.9)$, $td(w)=(0.7, 1)$
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\[ \text{td}(x) = (0.7, 1) \]

**Theorem 4.5.** Let \( G = (A, B) \) be a fuzzy graph and \( \mu_{A^+} \& \mu_{A^-} \) is a constant function. Then \( G \) is strongly irregular interval valued fuzzy graph if and only if \( G \) is a strongly total irregular interval valued fuzzy graph.

**Proof:** Let \( G = (A, B) \) be a fuzzy graph. And \( \mu_{A^+} \) are constant functions.

Let \( u_1, u_2, \ldots, u_n \) be the vertices of \( G \).

Let \( d^+(u_i) = k_i^+ \) and \( d^-(u_j) = k_j^- \) where \( k_i \neq k_j \).

Also \( \mu_{A^+}(u_i) = c_1, \mu_{A^-}(u_j) = c_2 \forall u_i \in V(G) \) where \( c_1, c_2 \) is constant, where \( c_i \in [0, 1] \)

Suppose \( G \) is strongly irregular interval valued fuzzy graph.

\[ \iff \forall u_i, u_j \in V(G) \]

\[ d^+(u_i) \neq d^+(u_j) \]

\[ \iff [k_i^+, k_i^-] \neq [k_j^-, k_j^+] \]

\[ \iff [k_i^- + c, k_i^+ + c] \neq [k_j^- + c, k_j^+ + c] \]

\[ \iff \text{td}(u_i) \neq \text{td}(u_j) \]

\[ \iff G \text{ is strongly totally irregular interval valued fuzzy graph.} \]

**Theorem 4.6.** If \( G = (A, B) \) is a strongly total irregular interval valued fuzzy graph then it is both highly total irregular interval valued fuzzy and neighbourly total interval valued irregular fuzzy graph.

**Proof:** Obvious

**Theorem 4.7.** Let \( G = (A, B) \) highly total irregular interval valued fuzzy and neighbourly total irregular interval valued fuzzy graph then \( G \) need not be a strongly total irregular interval valued fuzzy graph.

**Proof:** Proof similar as theorem 2.6

**Theorem 4.8.** A strongly irregular interval valued fuzzy graph need not be a strongly total irregular interval valued fuzzy graph.

**Proof:** Obvious, example 2.4 is strongly irregular but not strongly total interval valued fuzzy graph.

**Theorem 4.9.** Let \( G = (A, B) \) highly irregular total interval valued fuzzy and neighbourly total irregular interval valued fuzzy graph. If every pair of vertices in \( G \) is either adjacent or incident on the same vertex then \( G \) is strongly total irregular interval valued.

**Proof:** Obvious

**Theorem 4.10.** Let \( G = (A, B) \) be a fuzzy graph, where \( G^* \) is regular, \( \mu_{A^+}, \mu_{A^-} \) is a constant function and \( \mu_B^+(u, v) < \mu_A^+(u) \land \mu_A^-(v) \) for all \( u, v \in V(G) \). Then \( G \) is a strongly total irregular interval valued fuzzy graph iff
$G^c$ is a strongly total irregular interval valued fuzzy graph.

**Proof:** Let $G = (\sigma, \mu)$ be a strongly irregular fuzzy graph and $\sigma(u) = c$ for all $u \in G$.

\[ td^+(u) \neq td^+(v) \text{ for all } u, v \in V(G). \]

\[ \Leftrightarrow \mu_A(u) + \sum \mu_B(u, x_i) \neq \mu_A(v) + \sum \mu_B(v, y_j) \forall x_i \text{ incident on } u \forall y_j \text{ incident on } v \]

\[ \Leftrightarrow \mu_A(u) + \sum [c - \mu_B(u, x_i)] \neq \mu_A(v) + \sum [c - \mu_B(v, y_j)] \forall x_i \text{ incident on } u \forall y_j \text{ incident on } v, \text{ since } G^* \text{ is regular.} \]

\[ \Leftrightarrow \mu_A(u) + \sum [\mu_A(u) \land \mu_A(x_i) - \mu_B(u, x_i)] \neq \mu_A(v) + \sum [\mu_A(v) \land \mu_A(y_i) - \mu_B(v, y_i)] \forall x_i \text{ incident on } u \forall y_j \text{ incident on } v \]

\[ \Leftrightarrow \mu_A(u) + \sum \mu_B^c(u, x_i) \neq \mu_A(v) + \sum \mu_B^c(v, y_j) \forall u, v \in G^c. \]

Similarly we can prove for the negative degree.

**Theorem 4.11.** Let $H = (A', B')$ interval valued fuzzy subgraph of an irregular interval valued fuzzy graph $G = (A, B)$ irregular interval valued fuzzy graph where $G^*$ is regular, $\mu_A^+, \mu_A^-$ are constant function. Then $G$ is strongly total irregular iff $H$ is strongly total irregular interval valued.

**Proof:** Proof is similar as theorem 2.12

**Theorem 4.12.** The underlying crisp graph of a fuzzy graph $G = (A, B)$ is complete then $G$ is a neighbourly (or highly) total interval valued fuzzy graph if and only if $G$ is a strongly total interval valued irregular fuzzy graph.

**Proof:** Similar as theorem 2.13 and theorem 2.14

5. Domination in Irregular Fuzzy Graph:

The maximum degree of a graph $G$ is denoted by $\Delta(G)$.

**Definition 5.1.** A set $S \subseteq V$ in an irregular fuzzy graph $G=(A, B)$ is called a delta dominating($\Delta$-dominating) set if for every $u \in V-S$ there exists $v \in S$ such that $u$ and $v$ are adjacent in $G$ and $d(v) = \Delta(G)$.

A Dominating set $S$ is a minimal $\Delta$-dominating set if no proper subset of $S$ is a dominating set.

**Example 5.2.**
**Theorem 5.3.** If $G$ is a neighbourly irregular fuzzy graph and if $S$ is a $\Delta$-dominating set of $G$ then $V - S$ is not a dominating set.

**Proof:** Let $S$ be a dominating set of $G$.
Let $u$ and $v$ are adjacent in $G$ and $u \in S$, $v \in V - S$ and $d(u) = \Delta(G)$. Suppose $V - S$ is a dominating set then $d(v) = \Delta(G)$, $v \in V - S$.
Which contradicts the definition of neighbourly irregular.

**Theorem 5.4.** If $G$ is a strongly irregular fuzzy graph with $\Delta$-dominating set $S$ then $|S| = 1$.

**Proof:** Let $G$ is a strongly irregular fuzzy graph with $\Delta$-dominating set $S$, then no two vertices of $G$ are of same degree.
There exist only one vertex of degree $\Delta$ say $u$.
Since $G$ contains $S$, $u \in S$ dominates all other vertices of $G$.
Therefore $|S| = 1$.

**Theorem 5.5.** If $G$ is an irregular fuzzy graph if $S$ is a $\Delta$-dominating set of $G$ with $|S| > 1$ then $G$ is not strongly irregular.

**Proof:** Suppose $S$ $\Delta$-dominating set of $G$ and $|S| > 1$.
Then there exist at least two vertices in $S$ which dominates all the vertices in $V - S$ and $d(u_i) = \Delta$ for all $u_i \in S$.
That is there exist more than one vertex whose degree is equal to $\Delta$.
Which contradicts the definition of strongly irregular.

**Theorem 5.6.** Let $G$ is a strongly irregular fuzzy graph with $n + 1$ vertices and $S \subseteq V$ is a $\Delta$-dominating set then $K_{1,n}$ is a induced subgraph of $G^*$.

**Proof:** Let $G$ is a strongly irregular fuzzy graph with $n + 1$ vertices $S$. $\Delta$-dominating set then $|S| = 1 \Rightarrow |V - S| = n$.
Let $u \in S$, then $u$ dominates all the $n$ vertices of $V - S$, that is $u$ is adjacent to all the $n$ vertices of $V - S$.
Hence $K_{1,n}$ is the induced subgraph of $G^*$.

**Definition 5.7.** A set $S \subseteq V$ in an irregular interval valued fuzzy graph $G = (A, B)$ is called a $\Delta$-dominating set if for every $u \in V - S$ there exists $v \in S$
such that u and v are adjacent in G and \( d(v) = \Delta(G) \). \( d(v) = [d^-(v) = \Delta^-, d^+(v) = \Delta^+] \).

**Example 5.8.**

\[
\begin{array}{c}
\text{v}[.8, .9] \\
[.1, .2] \\
\text{u}[.8, .9] \\
[.2, .3] \\
\text{y}[.3, .7] \\
\text{w}[.4, .5] \\
\text{x}[.6, .7]
\end{array}
\]

\( d(u) = d(v) = [.5, .8] = [\Delta^-, \Delta^+] \). Here \( S = \{u, v\} \) is a \( \Delta \)-dominating set.

**Theorem 5.9.** If \( G \) is a neighbourly irregular interval valued fuzzy graph and if \( S \) is a \( \Delta \)-dominating set of \( G \) then \( V - S \) is not a dominating set.

**Proof:** Let \( S \) be a dominating set of \( G \). Let \( u \) and \( v \) are adjacent in \( G \) and \( u \in S, v \in V - S \) and \( d(u) = \Delta(G) \).

Suppose \( V - S \) is a dominating set then \( d(v) = \Delta(G), v \in V - S \)
Which contradicts the definition of neighbourly irregular.

**Theorem 5.10.** If \( G \) is a irregular interval valued fuzzy graph if \( S \) is a \( \Delta \)-dominating set of \( G \) with \( |S| > 1 \) then \( G \) is not strongly irregular.

**Proof:** Suppose \( S \) \( \Delta \)-dominating set of a interval valued fuzzy graph \( G \) and \( |S| > 1 \)
Then there exist atleast two vertices in \( S \) which dominates all the vertices in \( V - S \) and \( d(u) = \Delta \) for all \( u \in S \).
That is there exist more than one vertex whose degree is equal to \( \Delta \)
Which contradicts the definition of strongly irregular.

**Theorem 5.11.** Let \( G \) is a strongly irregular interval valued fuzzy graph with \( n+1 \) vertices and \( S \subseteq V \) is a \( \Delta \)-dominating set then \( K_{1,n} \) is a induced subgraph of \( G^* \).

**Proof:** Let \( G \) is a strongly irregular interval valued fuzzy graph with \( n + 1 \) vertices \( S \) .\( \Delta \)-dominating set then \( |S| = 1 \Rightarrow |V - S| = n \).
Let \( u \in S \), then \( u \) dominates all the \( n \) vertices of \( V - S \), that is \( u \) is adjacent to all the \( n \) vertices of \( V - S \) Hence \( K_{1,n} \) is the induced subgraph of \( G^* \).
6. Conclusion

In this paper we have described strongly irregular interval valued and strongly total irregular interval valued fuzzy graph. The necessary and sufficient conditions for an irregular interval valued to be the strongly irregular interval valued fuzzy graphs have been presented. Finally, Some results on delta-domination in irregular fuzzy graph and in irregular interval valued fuzzy graph have been proved.

References
