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Radiative Heat Transfer Effect on MHD Slip Flow of a Dissipating Nanofluid past an Exponential Stretching Porous Sheet

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Abstract

The present study examines the influence of velocity and thermal slip on MHD radiating and viscously dissipating flow of nanofluid past an exponentially stretching sheet. Using the similarity transformations, the governing boundary layer equations are transformed to set of ordinary differential equations and solved by Runge-kutta fourth order with shooting technique. The influence of various pertinent parameters on the velocity, temperature of the flow field and heat transfer characteristics are discussed. The present results are compared with the existing source literature and found an excellent agreement for the reduced cases.

AMS Subject Classification: 58D30,80A20,35K05,35Q79.

Key Words: MHD,Slip flow,Dissipation,Radiation,Exponential stretching sheet.

1 INTRODUCTION

Conventional heat transfer fluids like oil, water, and ethylene glycol mixtures, are poor heat transfer fluids because of their poor thermal conductivity. Many attempts have been taken by many researchers to enhance the thermal conductivity of these fluids by suspending nano/micro particles in liquids [1]. The characteristic feature of nanofluids is thermal conductivity enhancement, a phenomenon observed by many researchers[2]–[3]. Boundary-layer flow of nanofluid along with MHD and heat transfer over a linearly stretched surface have received a lot of attention because of its wide applications which include polymer extrusion, drawing of copper wires, continuous stretching of plastic films ,etc.,. A large number of researchers are engaged with this rich area. A benchmark of investigations was made over stretched surface of a boundary layer flow [4]–[5] .Radiative heat transfer plays a vital role in the area where high temperature prevails. A good literature on radiative transfer can be seen in well presented texts by Sparrow and Cess [6]. Arun Ishak [7] investigated MHD boundary layer flow due o an exponential stretching sheet with thermal radiation. On the other hand, in certain circumstances, the partial slip between the fluid and the moving surface may occur in situations when the fluid is particulate such as emulsions, suspensions, foam and polymer solutions. In these cases, the proper boundary condition is replaced by Navier's condition, where the amount of relative slip is proportional to local shear stress. Recently, Swati Mukhopadyay [8] analyzed the slip effects on MHD boundary layer flow over an exponential stretching sheet with suction/blowing and thermal radiation.

2 BASIC TRANSPORT EQUATIONS

A steady two dimensional laminar boundary layer flow of a viscous incompressible nanofluid past an exponentially stretching sheet coinciding with the plane $y = 0$ is considered. The flow is confirmed to $y > 0$ region. Keeping the origin fixed, two equal and opposite forces along the x - axis is applied so that the sheet is then stretched with velocity $u_w(x) = U_0 e^x/L$, where U_0 is constant. A variable magnetic field $B(x) = B_0 e^{x/2L}$ is applied normal to the sheet. Under the usual assumptions, the governing boundary layer equations are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_{nf}}\left\{\mu_{nf}\,\frac{\partial^2 u}{\partial y^2} - \sigma B^2 u\right\} \tag{2.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\mu_{nf}}{\rho Cp_{nf}}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho Cp_{nf}}\frac{\partial q_r}{\partial y} + \frac{\nu_{nf}}{\rho Cp_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.3}$$

The boundary conditions for the velocity and temperature fields are

$$u = u_w + N\nu_f\frac{\partial u}{\partial y}, v = -V_w, T = T_w + D\frac{\partial T}{\partial y}at \quad y = 0$$
$$u \longrightarrow 0, T \longrightarrow T_\infty \quad \text{as} \quad y \longrightarrow \infty \tag{2.4}$$

where u, v are the velocity components in the x and y directions, respectively, T - the temperature of the nanofluid, T_∞ - the ambient fluid temperature, σ - the electric conductivity, B_0 - the uniform magnetic field strength and q_r is the radiative heat flux. Here u_w is the stretching velocity, $T_w = T_\infty + T_0 e^{x/2L}$ is the temperature at the sheet, U_0, T_0 are the reference velocity and temperature respectively, $N = N_1 e^{-x/2L}$ is the velocity slip factor and $D = D_1 e^{-x/2L}$ is the thermal slip factor varies with x, D_1 is the initial value of thermal slip factor.

The effective density, thermal diffusivity, thermal expansion coefficient , effective dynamic viscosity and thermal conductivity of the nanofluid is given by

$$\rho_{nf} = (1-\phi)\,\rho_f + \phi\rho_s \quad, \alpha_{nf} = \frac{k_{nf}}{\rho Cp_{nf}}, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$$
$$\rho Cp_{nf} = (1-\phi)\,\rho Cp_f + \phi\rho Cp_s, \rho\beta_{nf} = (1-\phi)\,\rho\beta_f + \phi\rho\beta_s,$$
$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \tag{2.5}$$

where ϕ is the solid volume fraction of nanoparticles. By using the

Rosseland approximation, the radiative heat flux $q_r$is given by

$$q_r = -\frac{4}{3}\frac{\sigma^*}{k^*}\frac{\partial T^4}{\partial y} \tag{2.6}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Expanding 2.6 as Taylor series about and neglecting higher-order terms,we have $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$. In view of equations 2.6 and the above, equation 2.3 becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{\rho Cp_{nf}}(1+\frac{4}{3}R)\frac{\partial^2 T}{\partial y^2} + \frac{\nu_{nf}}{\rho Cp_{nf}}\Big(\frac{\partial u}{\partial y}\Big)^2 \tag{2.7}$$

Introducing the following similarity transformations

$$\eta = e^{\frac{x}{2L}} y\sqrt{\tfrac{U_0}{2\nu_f}}, \quad M = \sqrt{\tfrac{2\sigma B_0^2 L}{U_0\rho_f}}, \quad R = \tfrac{16\sigma * T_\infty^3}{3k_{nf}k^*}, \lambda = N_1\sqrt{\tfrac{U_0\nu_f}{2L}}$$
$$\mathrm{T}{=}\mathrm{T}_\infty + T_0 e^{\frac{x}{2L}}\theta(\eta), \quad Pr = \tfrac{\nu_f}{\alpha_f}, S = \tfrac{V_0}{\sqrt{\frac{U_0\nu_f}{2L}}}, \delta = D_1\sqrt{\tfrac{U_0}{2\nu_f L}}. \tag{2.8}$$

Equations 2.1, 2.2 and 2.7 take the following dimensionless form

$$f''' + (1-\phi)^{2.5}\left[\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\left(ff'' - f'^2\right) - Mf'\right] = 0 \tag{2.9}$$

$$\left(1+\frac{4}{3}R\right)\theta'' + Pr\left(\frac{k_f}{k_{nf}}\right) \times \left[\left(1-\phi+\phi\frac{\rho Cp_s}{\rho Cp_f}\right)\left(f\theta' - f'\theta\right) - \frac{Ec}{(1-\phi)^{2.5}}f''^2\right] = 0 \tag{2.10}$$

where prime denotes the differentiation with respect to η. The corresponding boundary conditions are

$$f'(0) = 1 + \lambda f''(0), \quad f(0) = S, \quad \theta(0) = 1 + \delta\theta'(0)$$
$$f'(\infty) \longrightarrow 0, \quad \theta(\infty) \longrightarrow 0 \tag{2.11}$$

The quantities of practical interest in this study are the skin friction or the shear stress coefficient C_f and the local Nusselt number Nu_x , the dimensionless quantities using similarity transformation are defined as

$$\sqrt{2ReC_f} = -\frac{1}{(1-\phi)^{2.5}}f''(0), \quad Nu_L = -\frac{Re}{2}\frac{k_{nf}}{k_f}\theta'(0) \tag{2.12}$$

where Re is the Reynolds number.

Table 1: Values of Nusselt number for different values of Prandtl number, Magnetic parameter and Radiation parameter with $\phi = \delta = \lambda = Ec = 0$.

Pr	M	R	Ishak[7]	Swathi Mukhopadyay [8]	Present Results
1	0	0	0.9548	0.9547	0.95485
2	0	0	1.4715	1.4714	1.47144
3	0	0	1.8691	1.8691	1.86906
5	0	0	2.5001	2.5001	2.50012
10	0	0	3.6604	3.6603	3.66037
1	0	0.5			0.67831
1	0	1	0.5312	0.5311	0.53132
1	1	0	0.8611	0.8610	0.86105
1	1	1	0.4505	0.4503	0.45032
2	0	0.5		1.0734	1.70352
2	0	1		0.8626	0.86330
3	0	0.5		1.3807	1.38071
3	0	1		1.1213	1.12141

3 RESULTS AND DISCUSSION

The governing boundary layer equations 2.9 and 2.10 subject to the boundary conditions 2.11 is solved numerically by employing Runge-Kutta method with shooting technique. In order to get a physical insight into the problem, a representative set of numerical results are depicted graphically in Figures 1-8. The present results are compared with that of Anuar Ishak [7] and Swathi Mukhopadyay [8] and found an excellent agreement for the reduced cases as displayed in Table 1.

It is interesting to observe that the effect of R is to raise the temperature distribution. Radiation increases the rate of energy transport to the fluid i.e., thermal conductivity of the nanofluid increases and hence the thermal boundary layer thickness increases, thereby increasing the temperature of the nanofluid Figure 1. Thus, the radiation can be used to control the thermal boundary layers quite effectively. As slip parameter increases, the velocity of the flow at the surface decreases because under the slip condition, the pulling of the stretching sheet can be only partly transmitted to the fluid and converges quickly Figure 2. Physically, when slip occurs, the flow velocity near the sheet is no longer equal to the stretching velocity of the sheet and the differences between the wall and the fluid velocities near to the wall rises. As a result, the hydrodynamic boundary layer thickness and hence the skin friction decreases. $f'(0) = 1$, when there is no slip.

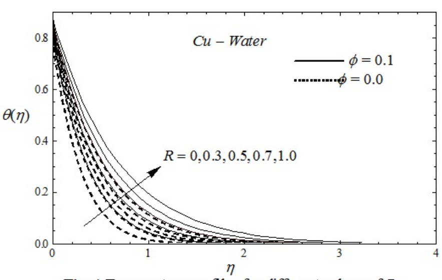

Fig. 1 Temperature profiles for different values of R

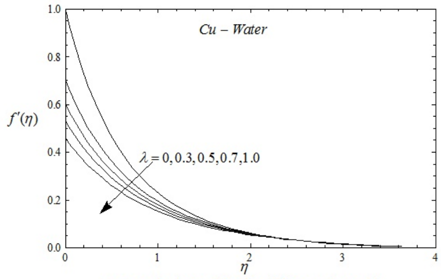

Fig. 2 Velocity profiles for different values of λ

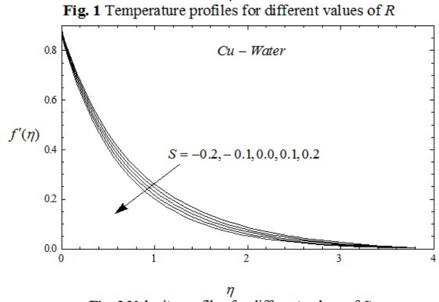

Fig. 3 Velocity profiles for different values of S

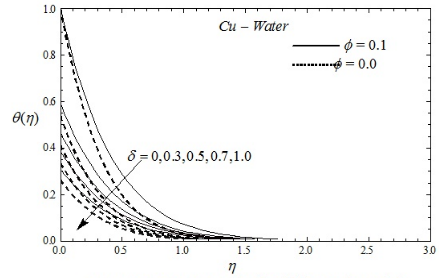

Fig. 4 Temperature profiles for different values of δ

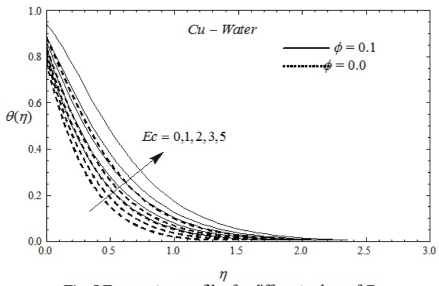

Fig. 5 Temperature profiles for different values of Ec

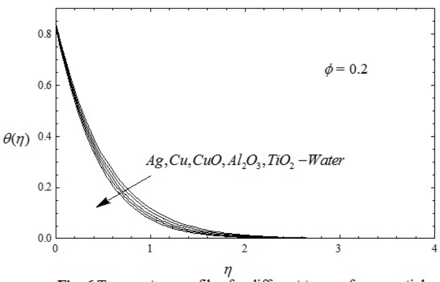

Fig. 6 Temperature profiles for different types of nanoparticles

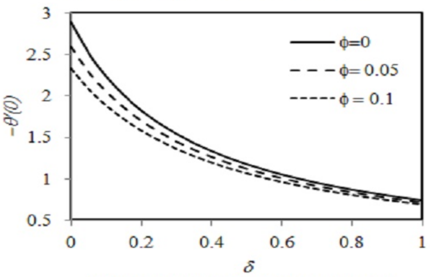

Fig. 7 Nusselt number for different values of δ

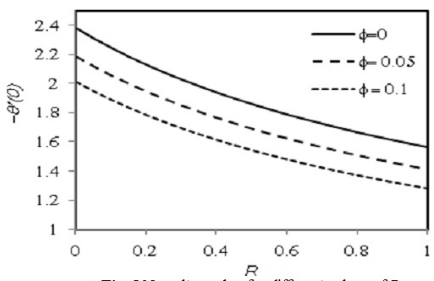

Fig. 8 Nusselt number for different values of R

From Figure 3 it is observed that the velocity decreases with an increase in the suction parameter. Physically, suction causes the boundary layer to adhere more closely to the wall and this destroys both momentum and thermal boundary layer leading a plunge in the nanofluid velocity and temperature. Injection adds nanofluid velocity via lateral mass flux through the sheet and this assists momentum layer development, causing an increase in momentum boundary layer thickness and a concomitant enhancement in the velocity. As the thermal slip parameterδ increases, less heat is transferred to the fluid from the sheet and so temperature is found to decrease Figure 4. The physical reason is that more flow will penetrate through the thermal boundary layer due to slip effect as δ increases. Hence more heat will be transferred and this will lead in the reduction of dimensionless surface temperature. Also it is observed that the temperature of the fluid is much higher for Cu-water than for regular fluid i.e.,$\phi = 0$.The upshot of different types of nanofluids with nanoparticle volume fraction ($\phi = 0.2$) containing Ag, Cu, Al_2O_3, CuO, TiO_2 as nanoparticles on temperature profile is portrayed in Figure 5. The thermal boundary layer increases from TiO2 to Ag. The positive Eckert number implies cooling of the sheet i.e., loss of heat from the sheet to the fluid. It is found that the translation velocity and temperature as well as thermal boundary layer thickness increase slightly with an increase in Ec Figure 6. Figure 7 shows the variations of temperature gradient versus thermal slip parameter $\delta$for different nanofluid volume fraction. It is evident that the rate of heat transfer decreases with an increase in the temperature jump parameter. It is evident that the rate of heat transfer decreases with an increase in ϕ. The effect of R on the heat transfer rate for different values of ϕ is portrayed in Figure 8.

4 Conclusions

From the above investigation, the following results can be summarized as follows: An increase in the nanofluid volume fraction parameter or radiation parameter or velocity slip parameter or Eckert number increases the fluid temperature while it drops off for an increase in S or δ. The local heat transfer rate decreases with an increase in the radiation parameter or thermal slip parameter or Eckert number.

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