ON SOFT SUPRA STRONGLY $b^*$-CLOSED SET IN SOFT SUPRA TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce the sets called a soft supra strongly $b^*$-closed set and soft supra strongly $b^*$-open set in a supra soft topological space $(X, \mu, E)$ and study the characterizations and their properties.

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1. Introduction

The concept of soft sets was first introduced by Molodtsov [1] in 1999 as a general mathematical tool for dealing with uncertain objects. In [2], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation
of the operations of soft sets, the properties and applications of soft set theory have been studied increasingly ([3], [4], [5]). In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets ([6], [7], [8], [9]). To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [10]. Recently, in 2011, Shabir and Naz [11] initiated the study of soft topological spaces. They defined soft topology on the collection $\tau$ of soft sets over $X$. Consequently, they defined basic notions of soft topological spaces such as open and closed soft sets, soft subspace, soft closure, soft nbd of a point and soft separation axioms, which is extended in [12]. In ([14], [15]), The researchers introduced some soft operations such as semi open soft, pre open soft, $\alpha$-open soft, $\beta$-open soft, $b$-open soft and investigated their properties in detail. Kandil et al. [16] introduced the notion of soft semi separation axioms. In particular, they study the properties of the soft semi regular spaces and soft semi normal spaces. In [17], Karal and Ahmed defined the notion of a mapping on classes of fuzzy soft sets, which is a fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets. Some fuzzy soft topological properties were introduced in ([18], [19], [20], [24]).

The notion of soft ideal was initiated for the first time by Kandil et al. [22]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, I)$. Applications to various fields were further investigated by Kandil et al. ([23], [24], [25], [26], [27], [28], [29]). In 1970, Levine [30] introduced the notion of $g$-closed sets in topological spaces as a generalization of closed sets. Recently, K. Kannan [31] introduced the concept of $g$-closed soft sets in a soft topological spaces. The notion of supra soft topological spaces was initiated for the first time by Elsheikh and Abd El-latief [32], which is extended in ([33], [14]). El-sheikh and Abd El-latief [32] have extended the notions of $g$-closed soft sets to such spaces. In this paper we introduce and study the concepts of soft supra $s-B^*-CS$ and soft supra $s-B^*-CS$ in supra soft topological spaces.

2. Preliminaries

In this section, we present the basic definitions and results of soft set theory.

**Definition 1.** [2] Let $X$ be an initial universe and $E$ be a set of parameters. Let $P(X)$ denote the power set of $X$ and $A$ be a non-empty subset of $E$. A pair
(\(F, A\)) denoted by \(F_A\) is called a soft set over \(X\), where \(F\) is a mapping given by \(F : A \rightarrow \mathcal{P}(X)\). In other words, a soft set over \(X\) is a parametrized family of subsets of the universe \(X\). For a particular \(e \in A\), \(F(e)\) may be considered the set of \(e\)-approximate elements of the soft set \((F, A)\) and if \(e \notin A\), then \(F(e) = \emptyset\) i.e., \(F_A = \{F(e) : e \in A \subseteq E, F : A \rightarrow \mathcal{P}(X)\}\), The family of all these soft sets denoted by \(SS(X)_A\).

**Definition 2.** [11] Let \(\tau\) be a collection of soft sets over a universe \(X\) with a fixed set of parameters \(E\), then \(\tau \subseteq SS(X)_A\) is called a soft topology on \(X\) if:

(a) \(\tilde{X}, \tilde{\phi} \in \tau\), where \(\tilde{\phi}(e) = \phi, \tilde{X}(e) = X, \forall e \in E\),

(b) the union of any number of soft sets in \(\tau\) belongs to \(\tau\),

(c) the intersection of any two soft sets in \(\tau\) belongs to \(\tau\).

The triplet \((X, \tau, E)\) is called a soft topological space over \(X\).

**Definition 3.** [34] Let \((X, \tau, E)\) be a soft topological space. A soft set \((F, A)\) over \(X\) is said to be closed soft set in \(X\), if its relative complement \((F, A)^c\) is open soft set.

**Definition 4.** [34] Let \((X, \tau, E)\) be a soft topological space. The members of are said to be open soft sets in \(X\). We denote the set of all open soft sets over \(X\) by \(OS(X, \tau, E)\), or when there can be no confusion by \(OS(X)\) and the set of all closed soft sets by \(CS(X, \tau, E)\) or \(CS(X)\).

**Definition 5.** [11] Let \((X, \tau, E)\) be a soft topological space, \((F, E) \in SS(X)_E\) and \(Y\) be a non-null subset of \(X\). Then the sub soft set of \((F, E)\) over \(Y\) denoted by \((F_Y, E)\), is defined as follows:

\[ F_Y(e) = Y \cap F(e) \quad \forall e \in E. \]

In other words \((F_Y, E) = \tilde{Y} \cap (F, E)\).

**Definition 6.** [11] Let \((X, \tau, E)\) be a soft topological space and \(Y\) be a non-null subset of \(X\). Then \(\tau_Y = \{(F_Y, E) : (F, E) \in \tau\}\) is said to be the soft relative topology on \(Y\) and \((Y, \tau_Y, E)\) is called a soft subspace of \((X, \tau, E)\).

**Definition 7.** [15] Let \((X, \tau, E)\) be a soft topological space and \((F, E) \in SS(X)_E\). Then \((F, E)\) is said to be semi open soft set if \((F, E) \subseteq cl(int(F, E))\). The set of all semi open soft sets is denoted by \(SOS(X)\) and the set of all semi closed soft sets is denoted by \(SOS(X)\).

**Definition 8.** [12] Let \((X, \tau_1, A)\) and \((Y, \tau_2, B)\) be soft topological spaces and \(f_{pu} : SS(X)_A \rightarrow SS(Y)_B\) be a function. Then, the function \(f_{pu}\) is called: (a) Continuous soft if \(f_{pu}^{-1}(G, B) \in \tau_1 \forall (G, B) \in \tau_2\). (b) Open soft if \(f_{pu}(G, A) \in \tau_2 \forall (G, A) \in \tau_1\). (b) Closed soft if \(f_{pu}(G, A) \in \tau_2 \forall (G, A) \in \tau_1\).
Theorem 9. [17] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets. For the soft function $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$, the following statements hold:

(a) $f_{pu}^{-1}((G, B)^c) = (f_{pu}^{-1}(G, B))^c \forall (G, B) \in SS(Y)_B$.

(b) $f_{pu}(f_{pu}^{-1}(G, B)) \subseteq (G, B) \forall (G, B) \in SS(Y)_B$. If $f_{pu}$ is surjective, then the equality holds.

(c) $(F, E) \subseteq f_{pu}^{-1}(f_{pu}((F, E))) \forall (F, A) \in SS(X)_A$. If $f_{pu}$ is injective, then the equality holds.

(d) $f_{pu}(\tilde{X}) \subseteq \tilde{Y}$.

(e) $f_{pu}^{-1}(\tilde{Y}) = \tilde{X}$ and $f_{pu}(\tilde{\phi}_A) = \tilde{\phi}_A$.

(f) If $(F, A) \subseteq (G, A)$, then $f_{pu}((F, A)) \subseteq f_{pu}((G, A))$.

(g) If $(F, B) \subseteq (G, B)$, then $f_{pu}^{-1}((F, B)) \subseteq f_{pu}^{-1}((G, B)) \forall (F, B), (G, B) \in SS(Y)_B$.

(h) $f_{pu}^{-1}([F, B] \upsilon (G, B)] = f_{pu}^{-1}((F, B)) \upsilon f_{pu}^{-1}((G, B))$ and $f_{pu}^{-1}([F, B]\cap (G, B)] = f_{pu}^{-1}((F, B))\cap f_{pu}^{-1}((G, B)) \forall (F, B), (G, B) \in SS(Y)_B$.

(i) $f_{pu}[(F, A) \cup (G, A)] = f_{pu}((F, A)) \cup f_{pu}((G, A))$ and $f_{pu}[(F, A) \cap (G, A)] = f_{pu}((F, A)) \cap f_{pu}((G, A)) \forall (F, A), (G, A) \in SS(X)_A$.

If $f_{pu}$ is injective, then the equality holds.

Definition 10. [31] A soft set $F_E \in SS(X, E)$ is called soft generalized closed in a soft topological space $(X, \tau, E)$ if $cl(F_E) \subseteq G_E$ whenever $F_E \subseteq G_E$ and $G_E \in \tau$.

Definition 11. [36] A soft set $F_E \in SS(X, E)$ is called soft strongly generalized closed in a soft topological space $(X, \tau, E)$ if $cl(int(F_E)) = G_E$ whenever $F_E \subseteq G_E$ and $G_E \in \tau$.

Definition 12. [27] A soft set $(F, E)$ is called soft regular closed set in a soft topological space $(X, \tau, E)$ if $cl(int(F, E)) = (F, E)$.

Definition 13. [37] A soft set $(F, E)$ is called soft regular generalized closed (soft rg-closed) in a soft topological space $(X, \tau, E)$ if $cl(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and $(G, E)$ is regular open soft in $X$. 

Definition 14. [1] Let $\tau$ be a collection of soft sets over a universe $X$ with a fixed set of parameters $E$, then $\mu \subseteq SS(X)_E$ is called supra soft topology on $X$ with a fixed set $E$ if (a) $\tilde{X}, \tilde{\phi} \in \mu$, (b) the union of any number of soft sets in $\mu$ belongs to $\mu$. The triplet $(X, \mu, E)$ is called supra soft topological space (or supra soft spaces) over $X$.

Definition 15. [29] A soft set $(F, E)$ is called soft supra generalized closed set (soft supra $g$-closed) in a supra soft topological space $(X, \mu, E)$ if $\text{cl}^s(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and $(G, E)$ is supra open soft in $X$.

Definition 16. [35] A soft set $(F, E)$ is called soft supra regular generalized closed (soft supra $rg$-closed) in a supra soft topological space $(X, \mu, E)$ if $\text{cl}^s(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and $(G, E)$ is supra regular open soft in $X$.

Definition 17. [19] A soft set $(F, E)$ of a supra soft topological space $(X, \mu, E)$ is called a soft supra semi-open set [briefly. soft supra SOS] if $(F, E) \subseteq \text{cl}^s(\text{int}^s((F, E)))$ and a soft supra semi-closed set [briefly. soft supra SCS] if $\text{int}^s(\text{cl}^s((F, E))) \subseteq (F, E)$.

3. Soft Supra Strongly $b^*$-Closed Set

Definition 18. A soft set $(F, E)$ of a soft topological space $(X, \mu, E)$ is called a supra soft $B$-open set [briefly. $s$-$BOS$] if

$$(F, E) \subseteq \text{cl}^s(\text{int}^s((F, E))) \cup \text{int}^s(\text{cl}^s((F, E)))$$

and the complement of a soft $B$-open set is called a supra soft $B$-closed set [briefly. $s$-$BCS$].

Definition 19. A soft set $(F, E)$ is called a soft supra strongly $b^*$-closed set [briefly. soft supra $s-B^*$-$CS$] in a supra soft topological space $(X, \mu, E)$ if $\text{cl}^s(\text{int}^s(F, E)) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and $(G, E)$ is soft $s$-$BOS$ in $X$.

Theorem 20. Let $(X, \mu, E)$ be a supra soft topological space and $A_E$ be a soft supra $s$-$BCS$ in $X$. If $A_E \subseteq H_E \subseteq \text{cl}^s(\text{int}^s(A_E))$, then $H_E$ is soft supra $s$-$BCS$.

Proof. Let $H_E \subseteq G_E$ and $G_E \in \mu$. Since $A_E \subseteq H_E \subseteq G_E$ and $A_E$ is soft supra...
s-BCS in $X$, then $cl^s(\text{int}^s(A_E)) \subseteq G_E$ implies $cl^s(\text{int}^s(H_E)) \subseteq cl^s(\text{int}^s(A_E)) \subseteq G_E$. Therefore, $H_E$ is soft supra s-BCS.

**Theorem 21.** A Soft subset $H_E$ of a supra soft topological space $(X, \mu, E)$ is soft supra s-B*-CS if and only if $cl^s(\text{int}^s(H_E))/H_E$ contains only null soft supra $s$-BCS.

**Proof.** *Necessity:* Let $H_E$ be a soft supra s-B*-CS, $F_E$ be a non null soft supra s-BCS and $F_E \subseteq cl^s(\text{int}^s(H_E))/H_E$. Then, $F_E \subseteq H_E^C$, implies that $H_E \subseteq F_E^C$. Since $H_E$ is soft supra s-B*-CS and $F_E^C$ is soft supra s-BOS, so $cl^s(\text{int}^s(H_E)) \subseteq F_E^C$. Hence, $F_E \subseteq [cl^s(\text{int}^s(H_E))]^c$. Therefore,

$$F_E \subseteq cl^s(\text{int}^s(H_E))^\cap [cl^s(\text{int}^s(H_E))]^c = \tilde{\phi}.$$  

Thus, $F_E = \tilde{\phi}$ which is a contradiction. Thus, $cl^s(\text{int}^s(H_E))/H_E$ contains only null soft supra s-BCS. *Sufficient:* Assume that $cl^s(\text{int}^s(H_E))/H_E$ contains only null soft supra s-BCS, $H_E \subseteq G_E$, $G_E$ is soft supra s-BOS and suppose that $cl^s(\text{int}^s(H_E))$ does not contained in $G_E$. Then, $cl^s(\text{int}^s(H_E))^\cap G_E^C$ is a non null supra $s$-B open soft subset of $cl^s(\text{int}^s(H_E))/H_E$, which is a contradiction. Thus, $H_E$ is a soft supra s-B*-CS.

**Corollary 22.** A soft supra s-B*-CS $H_E$ is supra soft regular closed if and only if $cl^s(\text{int}^s(H_E))/H_E$ is soft supra $s$-BCS and $H_E \subseteq cl^s(\text{int}^s(H_E))$.

**Proof.** *Necessity:* Sine $H_E$ is supra soft regular closed, then $cl^s(\text{int}^s(H_E)) = H_E$. Hence, $cl^s(\text{int}^s(H_E))/H_E = \tilde{\phi}$ is soft supra s-BCS. *Sufficient:* Assume that $cl^s(\text{int}^s(H_E))/H_E$ is soft supra s-BCS. Since $H_E$ is soft supra s-B*-CS, so $cl^s(\text{int}^s(H_E))/H_E$ contains only null soft supra s-BCS from the previous Theorem. Since $cl^s(\text{int}^s(H_E))/H_E$ is soft supra s-BCS, then it is soft supra s-B*-CS. It follows that, $cl^s(\text{int}^s(H_E))/H_E = \tilde{\phi}$, implies that $cl^s(\text{int}^s(H_E)) \subseteq H_E$. But, by hypothesis, $H_E \subseteq cl^s(\text{int}^s(H_E))$. Therefore, $cl^s(\text{int}^s(H_E)) = H_E$ and $H_E$ is supra soft regular closed.

**Theorem 23.** Let $(X, \mu, E)$ be a supra soft topological space and $F_E$ be a soft supra s-B*-CS in $X$. If $F_E \subseteq J_E \subseteq cl^s(\text{int}^s(F_E))$, then $J_E$ is soft supra s-B*-CS.

**Proof.** Let $J_E \subseteq G_E$ and $G_E \in \mu$. Since $F_E \subseteq J_E \subseteq G_E$ and $F_E$ is soft supra s-B*-CS in $X$, then $cl^s(\text{int}^s(F_E)) \subseteq G_E$ implies $cl^s(\text{int}^s(J_E)) \subseteq cl^s(\text{int}^s(F_E)) \subseteq G_E$. Therefore, $J_E$ is soft supra s-B*-CS.
Theorem 24. A Soft subset $H_E$ of a supra soft topological space $(X, \mu, E)$ is soft supra $s$-$B^*$-CS if $cl^s(int^s(H_E)) = H_E$ contains only null soft supra $s$-BCS.

Proof. Suppose that $F_E$ be a non null soft supra $s$-BCS and

$$F_E \subset cl^s(int^s(H_E)) = H_E.$$ 

Then, $F_E \subset H_E^c$, implies that $H_E \subset F_E^c$. Since $H_E$ is soft supra $s$-$B^*$-CS and $F_E$ is soft supra $s$-BCS, so $cl^s(int^s(H_E)) \subset F_E^c$. Hence,

$$F_E \subset [cl^s(int^s(H_E))]^c.$$ 

Therefore,

$$F_E \subset cl^s(int^s(H_E)) \cap [cl^s(int^s(H_E))]^c = \emptyset.$$ 

Thus, $F_E = \emptyset$ which is a contradiction. Thus, $cl^s(int^s(H_E)) = H_E$ contains only null soft supra $s$-BCS.

\[ \square \]

Theorem 25. A soft set $G_E$ is soft supra $s$-$B^*$-OS if and only if

$$F_E \subset cl^s(int^s(G_E))$$

whenever $F_E \subset G_E$ and $F_E$ is soft supra $s$-BOS.

Proof. Necessity: Let $F_E \subset G_E$ and $F_E$ is soft supra $s$-BOS in $X$, then $G_E^C \subset F_E^C$ and $F_E^C$ is soft supra $s$-BOS. Since $G_E^C$ is soft supra $s$-$B^*$-CS, so $cl^s(int^s(G_E^C)) \subset F_E^C$. Consequently, $F_E \subset [cl^s(int^s(H_E))]^c = int^s(cl^s(G_E))$. Sufficient: Let $G_E^C \subset H_E$ and $H_E$ is soft supra $s$-BOS in $X$. Then, $H_E^C \subset G_E$ and $H_E^C$ is soft supra $s$-BOS in $X$. Hence,

$$H_E^C \subset int^s(cl^s(G_E))$$

from the necessary condition. Thus, $[int^s(cl^s(H_E))]^c = cl^s(int^s(G_E^C)) \subset H_E$ and $H_E$ is soft supra $s$-BOS in $X$. This shows that, $G_E^C$ is soft supra $s$-$B^*$-CS in $X$. Therefore, $G_E$ is soft supra $s$-$B^*$-OS.

\[ \square \]

Theorem 26. If a soft subset $F_E$ of a supra soft topological space $(X, \mu, E)$ is both soft supra $s$-BCS and soft supra $s$-$B^*$-OS, then it is soft supra $s$-BOS.

Proof. Since $F_E$ is soft supra $s$-BCS and soft supra $s$-$B^*$-OS, then

$$F_E \subset [cl^s(F_E)] = int^s(F_E) \subset F_E.$$ 

Therefore, $F_E$ is soft supra $s$-BOS.

\[ \square \]
Corollary 27. If a soft subset $F_E$ of a supra soft topological space $(X, \mu, E)$ is both soft supra $s$-$BCS$ and soft supra $s$-$B^*-OS$, then it is both supra soft regular open and supra soft regular closed set.

Proof. Since $F_E$ is soft supra $s$-$BCS$ and soft supra $s$-$B^*-OS$. By the previous theorem, $F_E$ is soft supra $s$-$BOS$. Thus, \[ \text{int}^s(\text{cl}^s(F_E)) = F_E = \text{cl}^s(\text{int}^s(F_E)) \]
Therefore, $F_E$ is both supra soft regular open and supra soft regular closed set.

Corollary 28. If a soft subset $F_E$ of a supra soft topological space $(X, \mu, E)$ is both soft supra $s$-$BCS$ and soft supra $s$-$B^*-OS$, then it is supra soft $rg$-open set.

Proof. Obvious from the previous Theorem and Corollary.

Theorem 29. A soft subset $H_E$ of a supra soft topological space $(X, \mu, E)$ is soft supra $s$-$B^*-OS$ if and only if $H_E/\text{int}^s(\text{cl}^s(H_E))$ contains only null soft supra $s$-$BCS$.

Proof. It is similar to the proof of Theorem 3.4.

Theorem 30. Union of any two soft supra $s$-$BCS$ is soft supra $s$-$BCS$.

Proof. Suppose $(F, E)$ and $(H, E)$ are soft supra $s$-$BCS$ in $(X, \mu, E)$. Then $\text{cl}^s(F, E) \subseteq (G, E)$ and $\text{cl}^s(H, E) \subseteq (G, E)$ where $(F, E) \subseteq (G, E)$ and $(H, E) \subseteq (G, E)$. Hence \[ \text{cl}^s((F, E) \cup (H, E)) = \text{cl}^s(F, E) \cup \text{cl}^s(H, E) \subseteq (G, E). \]
That is $\text{cl}^s((F, E) \cup (H, E)) \subseteq (G, E)$ Therefore $(F, E) \cup (H, E)$ is a soft supra $s$-$BCS$ in $(X, \mu, E)$. In a similar way one can prove. Intersection of any two soft supra $s$-$BCS$ is a soft supra $s$-$BCS$.

Corollary 31. A soft supra $s$-$B^*$-$CS$ $H_E$ is supra soft regular closed if and only if $H_E/\text{int}^s(\text{cl}^s(H_E))$ is soft supra $s$-$BCS$ and $H_E \subseteq \text{int}^s(\text{cl}^s(H_E))$.

Proof. Follows from the previous Theorem.

Theorem 32. Let $(X, \mu, E)$ be a supra soft topological space and $F_E$ be a soft supra $s$-$B^*-OS$ in $X$. If $\text{int}^s(\text{cl}^s(F_E)) \subseteq H_E \subseteq F_E$, then $H_E$ is soft supra $s$-$B^*-OS$. 

Proof. Let $G_E \subseteq H_E$ and $G_E \in \mu_c$. Since $G_E \subseteq H_E \subseteq F_E$ and $F_E$ is soft supra $s-B^*-OS$ in $X$, then $G_E \subseteq \text{int}^s(\text{cl}^s(F_E))$. Hence,

$$G_E \subseteq \text{int}^s(\text{cl}^s(F_E)) \subseteq \text{int}^s(\text{cl}^s(H_E)).$$

Therefore, $H_E$ is soft supra $s-B^*-OS$.

**Theorem 33.** Union of any two soft supra $s-B^*-CS$ is soft supra $s-B^*-CS$.

Proof. Suppose $(F, E)$ and $(H, E)$ are soft supra $s-B^*-CS$ in $(X, \mu, E)$. Then $\text{cl}^s(F, E) \subseteq (G, E)$ and $\text{cl}^s(H, E) \subseteq (G, E)$ where $(F, E) \subseteq (G, E)$ and $(H, E) \subseteq (G, E)$ Hence

$$\text{cl}^s((F, E) \cup (H, E)) = \text{cl}^s(F, E) \cup \text{cl}^s(H, E) \subseteq (G, E).$$

That is $\text{cl}^s((F, E) \cup (H, E)) \subseteq (G, E)$ Therefore $(F, E) \cup (H, E)$ is a soft supra $s-B^*-CS$ in $(X, \mu, E)$. In a similar way one can prove. Intersection of any two soft supra $s-B^*-CS$ is a soft supra $s-B^*-CS$.

**References**


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