INTUITIONISTIC FUZZY $\alpha -$TRANSLATION
ON $\beta -$ALGEBRAS

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Abstract: In this paper, we define the concept of an Intuitionistic fuzzy $\alpha$-translation for some $\alpha \in [0, 1]$ on $\beta-$subalgebras of a $\beta-$algebra and investigate some of their elegant and simple results.

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1. Introduction

In 2002, J.Neggers and Kim [5] introduced $\beta-$algebras that arised from the classical and non-classical propositional logic. The theory of fuzzy sets proposed by L.A.Zadeh [7] in 1965 is generalized in 1986 by K.T.Aтанassov [2] as Intuitionistic fuzzy sets. Lee and Jun [3] applied the idea of fuzzy translation in BCK and BCI algebras. The fuzzy Translation was introduced in BF and BG algebras by M.Chandramouleswaran et. al [4]. In [1], the authors introduced the concept of fuzzy $\beta-$algebras. In [6], we introduced Intuitionistic fuzzy $\beta-$algebras and product in $\beta-$algebras. In this paper, we discuss the notion of Intuitionistic fuzzy Translation on $\beta-$algebras.
2. Preliminary

In this section we recall some basic definitions needed for our work.

**Definition 2.1.** A $\beta$-algebra is a non-empty set $X$ with a constant $0$ and two binary operations $+$ and $-$ satisfying the following axioms:

1. $x - 0 = x$
2. $(0 - x) + x = 0$
3. $(x - y) - z = x - (z + y) \ \forall \ x, y, z \in X.$

**Definition 2.2.** Let $A$ be a fuzzy subset of $X$ and $\alpha \in [0, 1 - \sup \{\mu_A(x)\}] \forall x \in X$. A mapping $(\mu_A)_\alpha^T : X \rightarrow [0, 1]$ is called Fuzzy $\alpha$-translation of $A$, if it satisfies $(\mu_A)^T_\alpha(x) = \mu_A(x) + \alpha \ \forall x \in X.$

**Definition 2.3.** Let $A$ be an Intuitionistic fuzzy subset of $X$ and $\alpha \in [0, 1 - \sup \{\mu_A(x) + \nu_A(x)\}] \forall x \in X$. A mapping $A^T_\alpha = \{(\mu_A)^T_\alpha, (\nu_A)^T_\alpha\}$ where $(\mu_A)^T_\alpha : X \rightarrow [0, 1]$ and $(\nu_A)^T_\alpha : X \rightarrow [0, 1]$ is called an Intuitionistic Fuzzy $\alpha$-translation of $A$, if it satisfies the following conditions

1. $(\mu_A)^T_\alpha(x + y) = \mu_A(x + y) + \alpha$
2. $(\nu_A)^T_\alpha(x + y) = \nu_A(x + y) - \alpha \ \forall x \in X.$

3. Intuitionistic Fuzzy $\alpha$-Translation on $\beta$-Algebras

In this section, we introduce the notion of Intuitionistic fuzzy $\alpha$-translation. To illustrate the concept, we discuss some examples. Also we prove some simple properties.

**Definition 3.1.** Let $X$ be a $\beta$-algebra. Let $A$ be an Intuitionistic fuzzy subset of $X$ and $\alpha \in [0, 1 - \sup \{\mu_A(x) + \nu_A(x)\}] \forall x \in X$. A mapping $A^T_\alpha = \{(\mu_A)^T_\alpha, (\nu_A)^T_\alpha\}$, where $(\mu_A)^T_\alpha : X \rightarrow [0, 1]$ and $(\nu_A)^T_\alpha : X \rightarrow [0, 1]$ is called an Intuitionistic Fuzzy $\alpha$-translation on $\beta$-subalgebra of $A$, if it satisfies the following conditions

1. $(\mu_A)^T_\alpha(x + y) = \mu_A(x + y) + \alpha$ and $(\mu_A)^T_\alpha(x - y) = \mu_A(x - y) + \alpha$
2. $(\nu_A)^T_\alpha(x + y) = \nu_A(x + y) - \alpha$ and $(\nu_A)^T_\alpha(x - y) = \nu_A(x - y) - \alpha \ \forall x \in X.$
Similarly, we can prove that \((\nu, \mu)\) are defined on \(X\) with the Cayley’s table

<table>
<thead>
<tr>
<th>+</th>
<th>0</th>
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<th>3</th>
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Now, \(A\) is defined as

\[
\mu_A(x) = \begin{cases} 
0.7 & x = 0,1 \\
0.6 & \text{otherwise}
\end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases} 
0.1 & x = 0,1 \\
0.3 & \text{otherwise}
\end{cases}
\]

Let us set \(\alpha = 0.05\), then

\[
A^T_\alpha = (\mu_A^T_\alpha(x), \nu_A^T_\alpha(x)),
\]

\[
(\mu_A^T_\alpha(x)) = \begin{cases} 
0.12 & x = 0,1 \\
0.11 & \text{otherwise}
\end{cases} \quad \text{and} \quad (\nu_A^T_\alpha(x)) = \begin{cases} 
0.05 & x = 0,1 \\
0.25 & \text{otherwise}
\end{cases}
\]

Hence \(A^T_\alpha\) is an Intuitionistic fuzzy \(\alpha\)-translation on \(\beta\)-subalgebra \(X\).

**Theorem 3.3.** If \(A\) is an IF \(\beta\)-subalgebra of \(X\), then \(A\) is an Intuitionistic fuzzy \(\alpha\)-translation of \(X\) for all \(\alpha \in \left[0, 1 - \sup \{\mu_A(x) + \nu_A(x)\}\right]\).

**Proof.** Let \(x, y \in X\) and \(\alpha \in \left[0, 1 - \sup \{\mu_A(x) + \nu_A(x)\}\right]\).

Then

\[
\mu_A(x + y) \geq \min \{\mu_A(x), \mu_A(y)\} \quad \text{and} \quad \nu_A(x + y) \leq \max \{\nu_A(x), \nu_A(y)\}.
\]

Now,

\[
(\mu_A^T_\alpha(x + y)) = \mu_A(x + y) + \alpha \\
\geq \min \{\mu_A(x), \mu_A(y)\} + \alpha \\
= \min \{\mu_A(x) + \alpha, \mu_A(y) + \alpha\} \\
= \min \{(\mu_A^T_\alpha(x)), (\mu_A^T_\alpha(y))\}
\]

Similarly, we can prove that \((\mu_A^T_\alpha(x - y)) \geq \min \{(\mu_A^T_\alpha(x)), (\mu_A^T_\alpha(y))\}\) and

\[
(\nu_A^T_\alpha(x + y)) = \nu_A(x + y) - \alpha \\
\leq \max \{\nu_A(x), \nu_A(y)\} - \alpha \\
= \max \{\nu_A(x) + \alpha, \nu_A(y) - \alpha\} \\
= \max \{(\nu_A^T_\alpha(x)), (\nu_A^T_\alpha(y))\}
\]

Similarly, we can prove that \((\nu_A^T_\alpha(x - y)) \leq \max \{(\nu_A^T_\alpha(x)), (\nu_A^T_\alpha(y))\}.

Hence \(A\) is an IF \(\alpha\)-translation \(\beta\)-subalgebra of \(X\).
Corollary 3.4. If \( A^T_\alpha = \{ (\mu_A)^T_\alpha, (\nu_A)^T_\alpha \} \) is an Intuitionistic Fuzzy \( \alpha \)-translation on \( \beta \)-subalgebra of \( \beta \)-algebra \( X \), then \( A \) is an IF\( \beta \)-subalgebra of \( X \).

One can easily prove the following results.

Lemma 3.5. If \( A^T_\alpha \) is an Intuitionistic Fuzzy \( \alpha \)-translation on \( \beta \)-subalgebra of \( \beta \)-algebra \( X \), then

1. \( (\mu_A)^T_\alpha(x) \leq (\mu_A)^T_\alpha(0) \)
2. \( (\nu_A)^T_\alpha(x) \geq (\nu_A)^T_\alpha(0) \)

Lemma 3.6. If \( A^T_\alpha \) is an Intuitionistic Fuzzy \( \alpha \)-translation on \( \beta \)-subalgebra of \( \beta \)-algebra \( X \), then

1. \( (\mu_A)^T_\alpha(x) \leq (\mu_A)^T_\alpha(x - 0) \)
2. \( (\nu_A)^T_\alpha(x) \geq (\nu_A)^T_\alpha(x - 0) \)

Definition 3.7. Let \( f : X \rightarrow Y \) be a function. Let \( A \) and \( B \) be two IF\( \alpha \)-translation on \( \beta \)-subalgebras in \( X \) and \( Y \) respectively. Then inverse image of \( B \) under \( f \) is defined by \( f^{-1}(B) = \{ f^{-1}(\mu_B)^T_\alpha(x), f^{-1}(\nu_B)^T_\alpha(x) | x \in X \} \) such that \( f^{-1}(\mu_B)^T_\alpha(x) = \mu_B(f(x) + \alpha) \) and \( f^{-1}(\nu_B)^T_\alpha(x) = \nu_B(f(x) - \alpha) \).

Theorem 3.8. Let \( X \) and \( Y \) be two \( \beta \)-algebras. Let \( A \) and \( B \) be two IF\( \alpha \)-translation on \( \beta \)-subalgebras. Let \( f : X \rightarrow Y \) be a homomorphism. If \( A \) is an IF\( \alpha \)-translation on \( \beta \)-subalgebra of \( Y \). Then \( f^{-1}(A) \) is a IF\( \alpha \)-translation on \( \beta \)-subalgebra of \( X \).

Proof. Let \( A \) be an IF\( \alpha \)-translation on \( \beta \)-subalgebra of \( Y \) and \( x, y \in Y \). Then

\[
\begin{align*}
  f^{-1}(\mu_A)^T_\alpha(x + y) &= f^{-1}(\mu_A)(x + y) + \alpha \\
  &= \mu_A(f(x + y)) + \alpha \\
  &= \mu_A(f(x) + f(y)) + \alpha \\
  &\geq \min \{ \mu_A(f(x) + \alpha), \mu_A(f(y) + \alpha) \} \\
  &= \min \{ f^{-1}(\mu_A)^T_\alpha(x), f^{-1}(\mu_A)^T_\alpha(y) \}.
\end{align*}
\]

Therefore

\[
  f^{-1}(\mu_A)^T_\alpha(x - y) \geq \min \{ f^{-1}(\mu_A)^T_\alpha(x), f^{-1}(\mu_A)^T_\alpha(y) \}.
\]

Similarly, we can prove that,

\[
  f^{-1}(\nu_A)^T_\alpha(x + y) \leq \max \{ f^{-1}(\nu_A)^T_\alpha(x), f^{-1}(\nu_A)^T_\alpha(y) \}.
\]
Moreover
\[ f^{-1}(\nu_A)^T_\alpha(x - y) \leq \max \{ f^{-1}(\nu_A)^T_\alpha(x), f^{-1}(\nu_A)^T_\alpha(y) \}. \]

Hence \( f^{-1}(A) \) is an IF\( \alpha \)-translation on \( \beta \)-subalgebra of \( X \).

**Theorem 3.9.** Let \( X \) and \( Y \) be two \( \beta \)-algebras. Let \( A \) and \( B \) be two IF\( \alpha \)-translation on \( \beta \)-subalgebras. Let \( f : X \to Y \) be a epimorphism. If \( A \) is an IF\( \alpha \)-translation on \( \beta \)-subalgebras of \( X \).Then \( f(A) \) is IF\( \alpha \)-translation on \( \beta \)-subalgebras of \( Y \).

That is \( f(\mu_B)^T_\alpha(x) = \mu_B(f(x) + \alpha) \) and \( f(\nu_B)^T_\alpha(x) = \nu_B(f(x) - \alpha) \).

**Proof.** Let \( A \) be an IF\( \alpha \)-translation on \( \beta \)-subalgebra of \( Y \) and \( x, y \in Y \).

Then
\[
\begin{align*}
  f(\mu_A)^T_\alpha(x + y) &= f(\mu_A(x + y) + \alpha) \\
  &= \mu_A(f(x + y) + \alpha) \\
  &= \mu_A(f(x) + f(y) + \alpha) \\
  &\geq \min \{ \mu_A(f(x) + \alpha), \mu_A(f(y) + \alpha) \} \\
  &= \min \{ f(\mu_A)^T_\alpha(x), f(\mu_A)^T_\alpha(y) \}
\end{align*}
\]

Therefore
\[
  f(\mu_A)^T_\alpha(x - y) \geq \min \{ f(\mu_A)^T_\alpha(x), f(\mu_A)^T_\alpha(y) \}.
\]

Similarly, we can prove that
\[
  f(\nu_A)^T_\alpha(x + y) \leq \max \{ f(\nu_A)^T_\alpha(x), f(\nu_A)^T_\alpha(y) \}.
\]

Moreover
\[
  f(\nu_A)^T_\alpha(x - y) \leq \max \{ f(\nu_A)^T_\alpha(x), f(\nu_A)^T_\alpha(y) \}.
\]

Hence \( f(A) \) is an IF\( \alpha \)-translation on \( \beta \)-subalgebra of \( Y \).

**References**


